GCE A LEVEL

1300U40-1

Z22-1300U40-1

TUESDAY, 21 JUNE 2022 – AFTERNOON

MATHEMATICS – A2 unit 4
APPLIED MATHEMATICS B

1 hour 45 minutes

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use gel pen or correction fluid. You may use a pencil for graphs and diagrams only.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the two boxes in the left-hand margin at the start of each answer, for example, | 0 | 1 | . Write the sub parts, e.g. **a**, **b** and **c**, within the white areas of the booklet.

Leave at least two line spaces between each answer.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 80.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

Section A: Statistics

0 1 An interview process involves three stages of selection. The first stage involves completing an aptitude test. The scores in the aptitude test are on a continuous scale and normally distributed with mean 66 and standard deviation 14. Candidates achieving the highest 5% of scores in the aptitude test progress immediately to the third stage of the interview process. Candidates achieving the lowest 5% of scores in the aptitude test do not progress any further in the interview process.

Ffion progresses to the second stage of the interview process. Find the range of possible scores that Ffion obtained in the aptitude test. [3]

0 2 System faults that occur in a particular type of aeroplane can be categorised in one of three mutually exclusive ways as compass (event C), gyroscopic (event G) or navigational (event N).

It has been established that, when a fault occurs, $\mathrm{P}(C) = 0{\cdot}4$, $\mathrm{P}(G) = 0{\cdot}35$ and $\mathrm{P}(N) = 0{\cdot}25$.

When a fault occurs, the cockpit shows one of three fault codes $F1$, $F2$ or $F3$.

It is known that:

$\mathrm{P}(F1 \mid C) = 0{\cdot}70$	$\mathrm{P}(F1 \mid G) = 0{\cdot}30$	$\mathrm{P}(F1 \mid N) = 0{\cdot}00$
$\mathrm{P}(F2 \mid C) = 0{\cdot}20$	$\mathrm{P}(F2 \mid G) = 0{\cdot}05$	$\mathrm{P}(F2 \mid N) = 0{\cdot}85$
$\mathrm{P}(F3 \mid C) = 0{\cdot}10$	$\mathrm{P}(F3 \mid G) = 0{\cdot}65$	$\mathrm{P}(F3 \mid N) = 0{\cdot}15$

The cockpit is showing a fault code. Calculate the probability that this aeroplane

a) displays the fault code $F1$, [2]

b) has a compass fault, given that it displays the fault code $F1$, [2]

c) has a gyroscopic fault, given that it does not display the fault code $F2$. [4]

0 3 Rectangles with perimeter 40 cm are produced randomly. The length, in cm, of the shorter side, X, is uniformly distributed across all possible values of X.

a) State the mean and variance of X. [3]

b) Find the probability that the area of a rectangle is greater than 96 cm^2. [5]

0 4 In a driving simulator, the stopping distances, in metres, for cars travelling at 20 mph and 30 mph can be modelled by normal distributions with means and standard deviations shown in the table below.

	Stopping distance (metres)	
Travelling at	**Mean**	**Standard deviation**
20 mph	12	3·5
30 mph	23	3·8

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a) Calculate the probability that a car travelling at 30 mph can stop within 30 metres. [2]

b) Suppose an obstacle suddenly appears 20 metres away. Dafydd states that you are 50 times as likely to collide with the obstacle if you are travelling at 30 mph as compared to 20 mph. Check whether Dafydd is correct. [4]

A campaigner for keeping the speed limit at 30 mph rather than reducing it to 20 mph in built-up areas claims that stopping distances are less than those stated above. He takes a random sample of 40 first-year university students and finds their mean stopping distance is 21·5 metres at 30 mph.

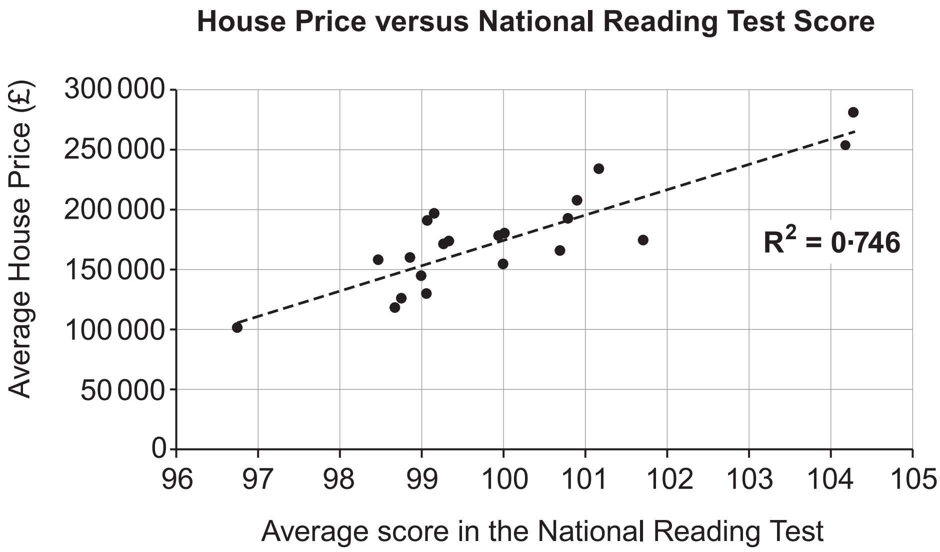
c) Test the campaigner's claim at the 1% level of significance. [6]

d) State a limitation to this test which calls into doubt the conclusion reached in part (c). [1]

TURN OVER

0 5 A government statistician wishes to investigate whether there is positive correlation between the variables *Average score in the National Reading Test* and *Average House Price*. She collects data on a randomly selected sample of 21 areas within Wales.

a) Carry out a 5% significance test for correlation between the two variables, using the computer output below. [5]



After seeing this output, the editor of a newspaper leads with the front-page headline,

"Buy a more expensive house to boost your children's reading ability!"

b) Discuss whether the data support the headline. [2]

c) Give a possible alternative explanation for this correlation. [1]

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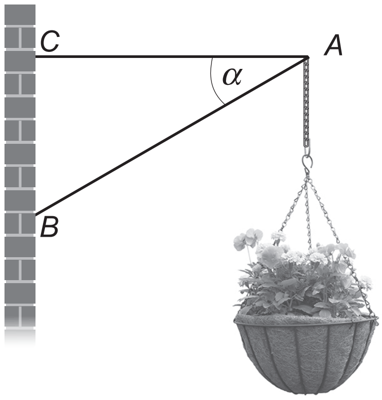
TURN OVER

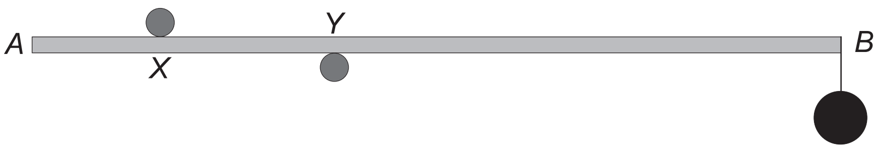
Section B: Differential Equations and Mechanics

0 6 The diagram shows a hanging basket attached at a point A to two light rods AB and AC. The rod AC is horizontal and the rod AB is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.



Given that the mass of the hanging basket is 3·6 kg, calculate the thrust in the rod AB and the tension in the rod AC. [5]

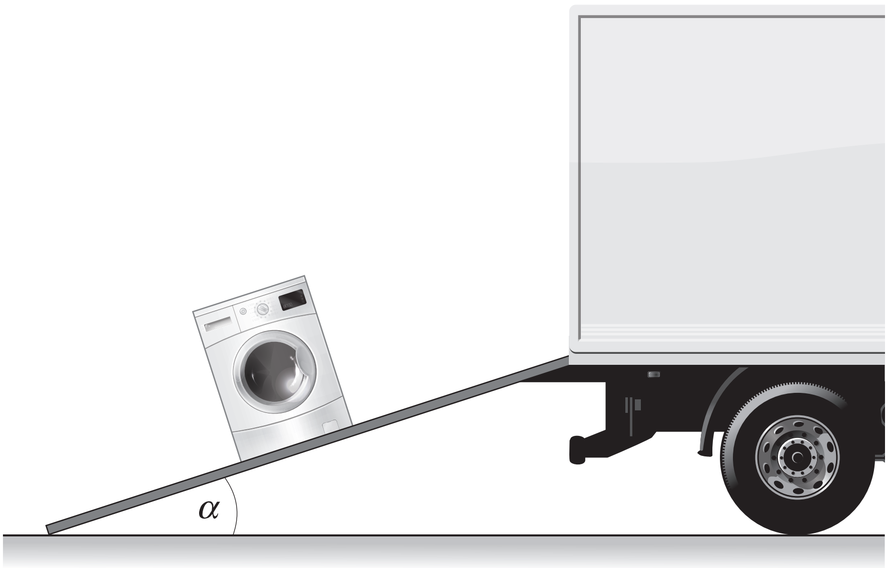
0 7 The diagram below shows a uniform rod AB, of length 2·4 m and mass 5 kg. The rod is held horizontally in equilibrium by means of two small, smooth, fixed cylindrical pegs at the points X and Y on the rod. The length of AX is 0·4 m and the length of XY is 0·5 m. An object of mass 11 kg is attached to the rod at point B. The force exerted on the rod by each of the pegs is vertical.



a) Find the magnitude of each of the forces exerted on the rod by the pegs at the points X and Y. Give your answers in terms of g. [7]

b) An additional object of mass M kg is attached at point A so that the rod is on the point of turning about Y. Calculate the value of M. [3]

0 8



The diagram shows a ramp being used to load a washing machine of mass 90 kg into a truck.

The washing machine may be modelled as a particle and the ramp as a rough plane inclined at an angle α to the horizontal. The coefficient of friction between the plane and the washing machine is $\frac{2}{9}$.

a) Suppose that the ramp is set up so that $\alpha = 10°$ and a rope is attached to the washing machine. When the rope is pulled with a constant force of 380 N parallel to the line of greatest slope of the ramp, the washing machine moves up the ramp. Calculate the acceleration of the washing machine. [5]

b) Suppose instead that, with the rope removed, the washing machine is at rest on the ramp. Calculate the maximum possible value of α. [3]

TURN OVER

0 9 Megan wants to make some ice cubes. She fills a plastic ice cube tray with water of temperature 10°C and places it directly into the freezer. The temperature inside the freezer remains constant at −18°C. The temperature of the water at time t hours after being placed in the freezer is denoted by θ°C. For Megan's scenario, Newton's law of cooling states that the rate of decrease of θ is directly proportional to the difference between the temperature of the water and the temperature inside the freezer.

a) Using k as the constant of proportionality, where k is positive, write down a differential equation satisfied by θ. [1]

b) Show that, for $\theta > -18$,

$$kt = \ln\left(\frac{28}{\theta+18}\right).$$

[4]

c) Given that the temperature of the water in the tray is 6°C after 1 hour, determine the total amount of time that Megan needs to wait for the temperature of the water to reach −5°C. Give your answer to the nearest hour. [3]

1 0 A golfer hits a ball towards a flag *F*, from a point *O*, with initial velocity of $35\,\text{ms}^{-1}$, at an angle θ above the horizontal. During its flight, the position vector of the golf ball relative to *O* is $(x\mathbf{i} + y\mathbf{j})$ metres, where the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical respectively.

a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = x\tan\theta - \frac{x^2}{250}(1+\tan^2\theta).$$

[4]

b) The ball just clears the top of a tree. The top of the tree has position vector $(100\mathbf{i}+20\mathbf{j})$ m and the base of the flag has position vector $(130\mathbf{i})$ m.

i) Determine the two possible values of $\tan\theta$.

ii) Given that the ball lands short of the flag *F*, find the shortest possible distance of the ball from the base of the flag when it first hits the ground. [5]

END OF PAPER