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## GCE MARKING SCHEME

## SUMMER 2017

MATHEMATICS - S3 0985-01

## INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

| Ques | Solution | Mark | Notes |
| :---: | :---: | :---: | :---: |
| 1 | $\begin{array}{\|c} \hline \bar{x}=59.1 \mathrm{si} \\ \text { Var estimate }=\frac{349425}{99}-\frac{5910^{2}}{100 \times 99}=1.4545 \ldots(16 / 11) \\ \text { (Accept division by } 100 \text { which gives } 1.44) \\ 99 \% \text { confidence limits are } \\ 59.1 \pm 2.576 \sqrt{1.4545 / 100} \\ \text { giving }[58.8,59.4] \text { cao } \end{array}$ | $\begin{gathered} \text { B1 } \\ \text { M1A1 } \\ \text { M1A1 } \\ \text { A1 } \end{gathered}$ | M0 if 100 or $\sqrt{ }$ omitted, A1 correct $z$ |
| 2(a) <br> (b) <br> (c) | Let $S$ denote the score on one of the dice. Then, $\mathrm{P}(S \leq x)=\frac{x}{6} \text { for } x=1,2,3,4,5,6$ <br> So $\begin{array}{r} \mathrm{P}(X \leq x)=\mathrm{P}(\text { All three scores } \leq x) \\ =\left(\frac{x}{6}\right)^{3} \\ P(X=x)=P(X \leq x)-P(X \leq x-1) \\ =\frac{x^{3}-(x-1)^{3}}{216}\left(\frac{3 x^{2}-3 x+1}{216}\right) \end{array}$ <br> A valid attempt at considering relevant probabilities. <br> Most likely value $=6$ | M1 <br> A1 <br> M1 <br> A1 <br> M1 <br> A1 | Convincing |
| 3 | $\begin{aligned} & \bar{x}=41.1 ; \bar{y}=34.9 \\ & s_{x}^{2}=\frac{84773}{49}-\frac{2055^{2}}{49 \times 50}=6.3775 \ldots(625 / 98) \\ & s_{y}^{2}=\frac{61121}{49}-\frac{1745^{2}}{49 \times 50}=4.5 \end{aligned}$ <br> [Accept division by 50 giving 6.25 and 4.41] $\begin{aligned} \mathrm{SE} & =\sqrt{\frac{6.3775 . .}{50}+\frac{4.5}{50}} \\ 0.4664 \ldots & (0.4617 \ldots) \\ z & =\frac{41.1-34.9-5}{0.4664 . .} \\ & =2.57(2.60) \\ p-\text {-value } & =0.005 \end{aligned}$ <br> Very strong evidence in support of Mair's belief (namely that the difference in the mean weights of male and female dogs is more than 5 kg ) | $\begin{gathered} \text { B1 } \\ \text { M1A1 } \\ \text { A1 } \\ \text { M1A1 } \\ \text { m1A1 } \\ \text { A1 } \\ \text { A1 } \\ \text { A1 } \end{gathered}$ | M0 no working <br> FT the $p$-value if less than 0.05 |

\begin{tabular}{|c|c|c|c|}
\hline Ques \& Solution \& Mark \& Notes <br>
\hline 4(a)

(b) \& | $\begin{aligned} & \hat{p}=0.32 \mathrm{si} \\ & \mathrm{ESE}=\sqrt{\frac{0.32 \times 0.68}{75}}(=0.05386 . .) \quad \mathrm{si} \\ & 95 \% \text { confidence limits are } \\ & 0.32 \pm 1.96 \times 0.05386 . . \\ & \text { giving }[0.21,0.43] \end{aligned}$ |
| :--- |
| The statement is incorrect because you cannot make a probability statement about a constant interval containing a constant value. |
| EITHER |
| The correct interpretation is that the calculated interval is an observed value of a random interval which contains the value of $p$ with probability 0.95 . |
| OR |
| If the process could be repeated a large number of times, then (approx) $95 \%$ of the intervals produced would contain $p$. | \&  \& M0 no working A1 correct $z$ <br>

\hline 5(a)

(b) \& | $\begin{aligned} \sum x & =306 ; \sum x^{2}=10407.52 \\ \text { UE of } \mu & =34 \\ \text { UE of } \sigma^{2} & =\frac{10407.52}{8}-\frac{306^{2}}{72} \\ & =0.44 \end{aligned} \quad \begin{aligned} \mathrm{DF} & =8 \mathrm{si} \end{aligned}$ $95 \% \text { confidence limits are }$ $34 \pm 2.306 \times \sqrt{\frac{0.44}{9}}$ |
| :--- |
| giving [33.5,34.5] cao | \& B1B1

B1
M1
A1
B1
B1
M1

A1 \& | No working need be seen |
| :--- |
| M0 division by 9 Answer only no marks |
| M0 for using $Z$ |
| FT from (a) | <br>

\hline
\end{tabular}



\begin{tabular}{|c|c|c|c|}
\hline Ques \& Solution \& Mark \& Notes \\
\hline 7(a)(i) \& \[
\begin{aligned}
E(X) \& =p+\frac{2(1-p)}{3}+\frac{3(1-p)}{3}+\frac{4(1-p)}{3} \\
\& =\frac{3 p+2-2 p+3-3 p+4-4 p}{3} \\
\& =3-2 p
\end{aligned}
\] \& \begin{tabular}{l}
M1 \\
A1 \\
A1
\end{tabular} \& \\
\hline (ii)

(b)(i) \& $$
\begin{aligned}
E\left(X^{2}\right)= & p+\left(2^{2}+3^{2}+4^{2}\right) \frac{(1-p)}{3} \\
\operatorname{Var}(X)=p & +\left(2^{2}+3^{2}+4^{2}\right) \frac{(1-p)}{3}-(3-2 p)^{2} \\
& =\frac{2}{3}+\frac{10}{3} p-4 p^{2} \\
& =\frac{2}{3}(1-p)(1+6 p)
\end{aligned}
$$ \& M1A1

A1

A1 \& $$
\left(\frac{29}{3}-\frac{26}{3} p\right)
$$ <br>

\hline \& | $\begin{aligned} E(U) & =\frac{3-E(X)}{2} \\ & =\frac{3-(3-2 p)}{2} \\ & =p \end{aligned}$ |
| :--- |
| (Therefore $U$ is an unbiased estimator) | \& M1

A1 \& M0 if no E <br>

\hline \& $$
\begin{aligned}
\operatorname{Var}(U) & =\frac{1}{4} \operatorname{Var}(\bar{X}) \\
& =\frac{\frac{2}{3}(1-p)(1+6 p)}{4 n}
\end{aligned}
$$ \& M1

A1 \& <br>
\hline (c)(i) \& $Y$ is $\mathrm{B}(n, p)$ \& B1 \& <br>

\hline \& | $\begin{aligned} \mathrm{E}(V) & =\frac{E(Y)}{n} \\ & =\frac{n p}{n}=p \end{aligned}$ |
| :--- |
| (Therefore $V$ is an unbiased estimator) | \& M1

A1 \& M0 if no E <br>

\hline (iii) \& $$
\begin{aligned}
\operatorname{Var}(V) & =\frac{\operatorname{Var}(Y)}{n^{2}} \\
& =\frac{p(1-p)}{n} \mathrm{oe}
\end{aligned}
$$ \& \[

$$
\begin{gathered}
\text { M1 } \\
\text { A1 }
\end{gathered}
$$
\] \& <br>

\hline
\end{tabular}

| Ques | Solution | Mark | Notes |
| :---: | :---: | :---: | :---: |
| (d) | $\operatorname{Var}(U)$ <br> $\operatorname{Var}(V)$$=\frac{\frac{2}{3}(1-p)(1+6 p)}{4 n} \div \frac{p(1-p)}{n}$ | M1 |  |
|  | $=\frac{1+6 p}{6 p}$ oe cao | A1 |  |
|  | $>1$ oe | A1 | No FT for incorrect ratio |
|  | Therefore $V$ is the better estimator. | A1 |  |
|  |  |  |  |

