GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - C3
0975/01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Mathematics C3 June 2017

Solutions and Mark Scheme

1.   (*a*)   5   3·258096538
                      5·5   3·442019376
                      6   3·610917913
                      6·5   3·766997233
                      7   3·912023005   (5 values correct)   **B2**

                     (**If B2 not awarded**, award B1 for either 3 or 4 values correct)

                     Correct formula with $h = 0·5$ **M1**

$$I \approx \frac{0·5}{3} \times \{3·258096538 + 3·912023005 + 4(3·442019376 + 3·766997233) + 2(3·610917913)\}$$

$$I \approx 43·22802181 \times 0·5 \div 3$$

$$I \approx 7·204670301$$

$$I \approx 7·2 \qquad \text{(f.t. one slip)} \qquad \textbf{A1}$$

                     Note: Answer only with no working earns 0 marks

        (*b*)

$$\int_5^7 \ln\left(\frac{3}{\sqrt{(1+x^2)}}\right) dx = \int_5^7 \ln 3 \, dx - \frac{1}{2}\int_5^7 \ln(1+x^2)\, dx \qquad \textbf{M1}$$

$$\frac{1}{2}\int_5^7 \ln(1+x^2)\, dx \approx 3·6 \qquad \text{(f.t. candidate's answer to (\textit{a}))} \qquad \textbf{B1}$$

$$\int_5^7 \ln\left(\frac{3}{\sqrt{(1+x^2)}}\right) dx \approx 2·2 - 3·6 = -1·4$$
$$\text{(f.t. candidate's answer to (\textit{a}))} \qquad \textbf{A1}$$

2. (*a*) $6(\sec^2\theta - 1) - 6 = 4\sec^2\theta + 5\sec\theta.$

(correct use of $\tan^2\theta = \sec^2\theta - 1$)   M1

An attempt to collect terms, form and solve quadratic equation in $\sec\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sec\theta + b)(c\sec\theta + d)$, with $a \times c$ = candidate's coefficient of $\sec^2\theta$ and $b \times d$ = candidate's constant   m1

$2\sec^2\theta - 5\sec\theta - 12 = 0 \Rightarrow (2\sec\theta + 3)(\sec\theta - 4) = 0$

$\Rightarrow \sec\theta = -\dfrac{3}{2}$, $\sec\theta = 4$

$\Rightarrow \cos\theta = -\dfrac{2}{3}$, $\cos\theta = \dfrac{1}{4}$ (c.a.o.)   A1

$\theta = 131\cdot81°, 228\cdot19°$ B1 B1

$\theta = 75\cdot52°, 284\cdot48°$ B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\cos\theta$ = +, −, f.t. for 3 marks, $\cos\theta$ = −, −, f.t. for 2 marks

$\cos\theta$ = +, +, f.t. for 1 mark

(*b*)   Correct use of $\sec\phi = \dfrac{1}{\cos\phi}$ and $\tan\phi = \dfrac{\sin\phi}{\cos\phi}$ (o.e.)   M1

$\sin\phi = -\dfrac{3}{5}$ A1

$\phi = 323\cdot13°, 216\cdot87°$ (f.t. for $\sin\phi = -a$)   A1

3. (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}(2y^3) = 6y^2\,\dfrac{\mathrm{d}y}{\mathrm{d}x}$ B1

   $\dfrac{\mathrm{d}}{\mathrm{d}x}(-3x^2y) = -3x^2\,\dfrac{\mathrm{d}y}{\mathrm{d}x} - 6xy$ B1

   $\dfrac{\mathrm{d}}{\mathrm{d}x}(x^4) = 4x^3,\ \dfrac{\mathrm{d}}{\mathrm{d}x}(-4x) = -4,\ \dfrac{\mathrm{d}}{\mathrm{d}x}(7) = 0$ B1

   $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4 - 4x^3 + 6xy}{6y^2 - 3x^2}$ (o.e.)   (c.a.o.)   B1

   (b)   (i)   candidate's x-derivative $= 7 + 4t$ B1

   candidate's y-derivative $= \dfrac{(7 + 4t)r - (4 + 3t)m}{(7 + 4t)^2}$,

   where r, m are integers   M1

   candidate's y-derivative $= \dfrac{(7 + 4t)3 - (4 + 3t)4}{(7 + 4t)^2}$ A1

   $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

   $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{5}{(7 + 4t)^3}$ (c.a.o.)   A1

   (ii) $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) = \dfrac{-3 \times 5 \times 4}{(7 + 4t)^4}$ (o.e.)

   (f.t. candidate's expression of correct given form for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$)   B1

   Use of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) \div \dfrac{\mathrm{d}x}{\mathrm{d}t}$

   (f.t. candidate's expression for $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$)   M1

   $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{-60}{(7 + 4t)^5}$ (c.a.o.)   A1

4. (a) (i) $V(x) = 150 \Rightarrow x \times (x + 4) \times (x - 2) = 150$ M1
$x^3 + 2x^2 - 8x - 150 = 0$ (convincing)   A1

(ii)   Let $f(x) = x^3 + 2x^2 - 8x - 150$
Use of a correct method to find $f(x)$ when $x = 5$ and $x = 6$ M1
$f(5) = -15 \ (< 0), f(6) = 90 \ (> 0)$
Change of sign $\Rightarrow 5 < x < 6$ A1

(b) $x_0 = 6$
$x_1 = 5 \cdot 013297935$ (x_1 correct, at least 2 places after the point)   B1
$x_2 = 5 \cdot 190516135$
$x_3 = 5 \cdot 163166906$
$x_4 = 5 \cdot 167508826 = 5 \cdot 17$ (x_4 correct to 2 decimal places)   B1
An attempt to check values or signs of $f(x)$ at $x = 5 \cdot 165$ and
$x = 5 \cdot 175$ M1
$f(5 \cdot 165) = -0 \cdot 178 \ (< 0), f(5 \cdot 175) = 0 \cdot 751 \ (> 0)$ A1
Change of sign $\Rightarrow x = 5 \cdot 17$ correct to two decimal places   A1

5. (a) (i) $\dfrac{dy}{dx} = \dfrac{1}{2} \times (3x^2 + 5x)^{-1/2} \times f(x)$ $(f(x) \neq 1)$ M1

$\dfrac{dy}{dx} = \dfrac{1}{2} \times (3x^2 + 5x)^{-1/2} \times (6x + 5)$ A1

(ii) $\dfrac{dy}{dx} = \dfrac{\mathbf{3}}{\mathbf{\sqrt{(1 - (3x)^2)}}}$ **or** $\dfrac{1}{\sqrt{(1 - (3x)^2)}}$ **or** $\dfrac{3}{\sqrt{(1 - 3x^2)}}$ M1

$\dfrac{dy}{dx} = \dfrac{3}{\sqrt{(1 - 9x^2)}}$ A1

(b) $x = \cot y \Rightarrow \dfrac{dx}{dy} = -\operatorname{cosec}^2 y$ B1

$\dfrac{dx}{dy} = -(1 + \cot^2 y)$ B1

$\dfrac{dx}{dy} = -(1 + x^2)$ B1

$\dfrac{dy}{dx} = -\dfrac{1}{1 + x^2}$ (c.a.o.)   B1

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6. (*a*) (i) $\int 8e^{2-5x}dx = k \times 8 \times e^{2-5x} + c$ $(k = 1, -5, {}^1/_5, -\mathbf{{}^1\!/_5})$ M1

$\int 8e^{2-5x}dx = -\dfrac{8}{5} \times e^{2-5x} + c$ A1

(ii) $\int 6(4x-7)^{-1/3} = \dfrac{6 \times k \times (4x-7)^{2/3}}{2/3} + c$ $(k = 1, 4, \mathbf{{}^1\!/_4})$ M1

$\int 6(4x-7)^{-1/3} = \dfrac{6 \times 1/4 \times (4x-7)^{2/3}}{2/3} + c$

$\int 6(4x-7)^{-1/3} = \dfrac{9}{4} \times (4x-7)^{2/3} + c$ A1

(iii) $\int \cos\left[\dfrac{7x-9}{3}\right] dx = k \times \sin\left[\dfrac{7x-9}{3}\right] + c$ $(k = 1, {}^7/_3, \mathbf{{}^3\!/_7}, -{}^3/_7, {}^1/_7)$ M1

$\int \cos\left[\dfrac{7x-9}{3}\right] dx = \dfrac{3}{7} \times \sin\left[\dfrac{7x-9}{3}\right] + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) (i) $\dfrac{dy}{dx} = \dfrac{a+bx}{3x^2-8}$ (including $a = 1, b = 0$)   M1

$\dfrac{dy}{dx} = \dfrac{6x}{3x^2-8}$ A1

(ii) $\displaystyle\int_2^6 \dfrac{3x}{3x^2-8} dx = r\left[\ln(3x^2-8)\right]_2^6$

where r is a constant   M1

$\displaystyle\int_2^6 \dfrac{3x}{3x^2-8} dx = \dfrac{1}{2}\left[\ln(3x^2-8)\right]_2^6$ A1

$\displaystyle\int_2^6 \dfrac{3x}{3x^2-8} dx = r\{\ln(108-8)) - \ln(12-8)\}$ m1

$\displaystyle\int_2^6 \dfrac{3x}{3x^2-8} dx = \ln(5)$ (c.a.o.)   A1

7. (*a*)   Choice of negative x M1

                Correct verification that L.H.S. of inequality > 5 and a statement to the
                effect that this is in fact the case   A1

         (*b*) $a = -\dfrac{1}{2}$ B1

                     $b = -6$ B1

8. (*a*) $y - 2 = \dfrac{3}{\sqrt{(5x - 4)}}$ B1

                An attempt to isolate $5x - 4$ by crossmultiplying and squaring   M1

                $x = \dfrac{1}{5}\left[4 + \dfrac{9}{(y-2)^2}\right]$ (c.a.o.)   A1

                $f^{-1}(x) = \dfrac{1}{5}\left[4 + \dfrac{9}{(x-2)^2}\right]$

                               (f.t. one slip in candidate's expression for x)   A1

         (*b*) $D(f^{-1}) = (2,\, 2{\cdot}5]$ B1 B1

9. (*a*) $R(f) = [8 + k,\, \infty)$ B1

         (*b*) $8 + k \geq -3$ M1

                $k \geq -11$ $(\Rightarrow$ least value of k is $-11)$

                                 (f.t. candidate's $R(f)$ provided it is of form $[a, \infty)$)   A1

         (*c*)   (i) $gf(x) = (4x + k)^2 - 9$ B1

                    (ii) $(4 \times 2 + k)^2 - 9 = 7$

                          (substituting 2 for x in candidate's expression for $gf(x)$

                                         and putting equal to 7)   M1

                          Either $k^2 + 16k + 48 = 0$ or $(8 + k)^2 = 16$ (c.a.o.)   A1

                          $k = -4,\, -12$ (f.t. candidate's quadratic in k)   A1

                          $k = -4$ (c.a.o.)   A1