# GCE AS/A Level

0982/01

S17-0982-01

# MATHEMATICS – M3
## Mechanics

FRIDAY, 23 JUNE 2017 – MORNING

1 hour 30 minutes

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Take $g$ as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** A particle moves along the $x$-axis such that its displacement $x$ metres at time $t$ seconds satisfies the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} + x = 2.$$

The particle passes through the origin when $t = 0$.

*(a)* Find the time when the particle reaches the point $x = 1$, and determine an expression for $x$ at time $t$. [7]

*(b)* Hence find an expression for the acceleration of the particle at time $t$. [3]

**2.** Two particles *P* and *Q*, of mass 3 kg and 7 kg respectively, are attached one to each end of a light inextensible string. Initially, the string is slack and the particles are at rest on a smooth horizontal surface. The particle *Q* is then projected across the surface with speed 8 ms$^{-1}$ away from *P* along the straight line passing through the initial positions of *P* and *Q*. Find the speed with which the particles begin to move immediately after the jerk and determine the impulsive tension in the string during the jerk. [6]

**3.** The function $x$ satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 6\frac{\mathrm{d}x}{\mathrm{d}t} + (10-k)x = \frac{1}{50}k(k-5)(12t-26),$$

where $k$ is a constant. When $t = 0$, $x = 8$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 16$. Find $x$ in each of the following cases.

*(a)* $k = 5$. [5]

*(b)* $k = 0$. [5]

*(c)* $k = 10$. [8]

**4.** An object *P*, of mass 0·5 kg, moves along a horizontal straight line. The object experiences a resistive force of magnitude $3v^2$ N, where $v$ ms$^{-1}$ is the speed of *P* at time $t$ seconds. When $t = 0$, *P* is at a point *O* and moving with speed 2 ms$^{-1}$.

*(a)* Show that $v$ satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -6v^2.$$ [2]

*(b)* Find an expression for $v$ in terms of $t$. [4]

*(c)* Obtain an expression for $v$ in terms of $x$, where $x$ metres is the distance of *P* from *O* at time $t$ seconds. [5]

*(d)* Determine, in terms of $x$, the rate at which work is being done against the resistance when *P* is at a distance $x$ metres from *O*. [3]

 

**5.** The speed $v$ ms$^{-1}$ of a particle moving along the $x$-axis is given by

$$v^2 = -4x^2 + 8x + 21.$$

*(a)* Show that the motion is simple harmonic and write down the centre of the motion. [5]

*(b)* Show that the period of the motion is $\pi$ seconds and determine the amplitude. [4]

*(c)* Given that when $t = 0$, the particle is at the centre of the motion and moving with positive velocity, write down an expression for $x$ in terms of $t$ and calculate the time taken for the particle to reach $x = 3$ for the first time. [4]

**6.** A ladder $AB$, of length 8 m and weight $W$N, rests with one end $A$ against a vertical wall and the other end $B$ on horizontal ground. The ladder makes an angle $\alpha$ with the horizontal where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the ladder and the wall is $\lambda$ and the coefficient of friction between the ladder and the ground is $\mu$.

*(a)* Consider the case when the ladder is **uniform**. Given that $\lambda = 0$ and the ladder is on the point of slipping, determine the value of $\mu$ in this case. [4]

*(b)* Consider the case when the ladder is **non-uniform** and its centre of mass is $x$ m from $A$. Given that $\lambda = \mu = 0{\cdot}6$ and the ladder is on the point of slipping, calculate the value of $x$ in this case. [10]

**END OF PAPER**

 