GCE MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced

SUMMER 2015

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2015 examination in GCE MATHEMATICS - C1-C4 & FP1-FP3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

1. (*a*) (i) Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -\frac{1}{3}$ (or equivalent) A1

(ii) A correct method for finding the equation of AB using candidate's gradient for AB M1

Equation of AB : $y - 3 = -\frac{1}{3}[x - (-7)]$ (or equivalent)

(f.t. candidate's gradient of AB) A1

Equation of AB : $x + 3y - 2 = 0$ (convincing) A1

(iii) Use of $m_L \times m_{AB} = -1$ M1

A correct method for finding the equation of L using candidate's gradient for L (M1)

(to be awarded only if corresponding M1 is not awarded in part (ii))

Equation of L : $y - 5 = 3[x - (-3)]$ (or equivalent)

(f.t. candidate's gradient of AB) A1

Note: Total mark for part (*a*) is 7 marks

(*b*) An attempt to solve equations of AB and L simultaneously M1

$x = -4$, $y = 2$ (convincing) (c.a.o.) A1

(*c*) A correct method for finding at least one coordinate of the mid-point of AB M1

y-coordinate of the mid-point of $AB = 1{\cdot}5$ (or x-coordinate $= -2{\cdot}5$)

$\Rightarrow$ D is not the mid-point of AB **or**

$\Rightarrow$ L is not the perpendicular bisector of AB **or**

$\Rightarrow$ the mid-point does not lie on L A1

Alternative Mark Scheme

A correct method for finding the lengths of two of AB, AD, BD M1

Two of $AB = \sqrt{90}$, $AD = \sqrt{10}$, $BD = \sqrt{40}$

$\Rightarrow$ D is not the mid-point of AB **or**

$\Rightarrow$ L is not the perpendicular bisector of AB **or**

$\Rightarrow$ the mid-point does not lie on L A1 (*d*) A correct method for finding the length of $BD(CD)$ M1

$BD = \sqrt{40}$ (or equivalent) A1

$CD = \sqrt{10}$ A1

Substitution of candidate's derived values in $\tan ABC = \frac{CD}{BD}$ m1

$\tan ABC = \frac{1}{2}$ (c.a.o.) A1

Special Case

A candidate who has been awarded M0 A0 A0 m0 A0 may be awarded SC1 for one of $AB = \sqrt{90}$, $AC = \sqrt{20}$, $BC = \sqrt{50}$

2. (*a*) $\dfrac{4\sqrt{2} - \sqrt{11}}{3\sqrt{2} + \sqrt{11}} = \dfrac{(4\sqrt{2} - \sqrt{11})(3\sqrt{2} - \sqrt{11})}{(3\sqrt{2} + \sqrt{11})(3\sqrt{2} - \sqrt{11})}$ M1

Numerator: $12 \times 2 - 4 \times \sqrt{2} \times \sqrt{11} - 3 \times \sqrt{11} \times \sqrt{2} + 11$ A1

Denominator: $18 - 11$ A1

$\dfrac{4\sqrt{2} - \sqrt{11}}{3\sqrt{2} + \sqrt{11}} = 5 - \sqrt{22}$ (c.a.o.) A1

Special case

If M1 not gained, allow SC1 for correctly simplified numerator or denominator following multiplication of top and bottom by $3\sqrt{2} + \sqrt{11}$

(*b*) $\dfrac{7}{2\sqrt{14}} = p\sqrt{14}$, where p is a fraction equivalent to $^{1}/_{4}$ B1

$\left(\dfrac{\sqrt{14}}{2}\right)^3 = q\sqrt{14}$, where q is a fraction equivalent to $^{7}/_{4}$ B1

$\dfrac{7}{2\sqrt{14}} + \left(\dfrac{\sqrt{14}}{2}\right)^3 = 2\sqrt{14}$ (c.a.o.) B1

3. (*a*) y-coordinate of $P = -4$ B1

$\frac{dy}{dx} = 3x^2 - 2x - 13$

(an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 2$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = -5$ (c.a.o.) A1

Gradient of normal = $\frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

Equation of normal to C at P: $y - (-4) = {}^1/_5(x - 2)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ and the candidate's derived y-value at $x = 2$ provided M1 and both m1's awarded) A1

(*b*) Putting candidate's expression for $\frac{dy}{dx} = -8$ M1

An attempt to collect terms, form and solve quadratic equation in a (or x) either by correct use of the quadratic formula or by getting the equation into the form $(ma + n)(pa + q) = 0$, with $m \times p$ = candidate's coefficient of a^2 and $n \times q$ = candidate's constant m1

$3a^2 - 2a - 5 = 0 \Rightarrow a = -1$ or $\frac{5}{3}$ (both values) (c.a.o.) A1

4. (*a*) $4(x - 3)^2 - 225$ B1 B1 B1

(*b*) $4(x - 3)^2 = 225$ (f.t. candidate's values for a, b, c) M1

$(x - 3) = (\pm)\frac{15}{2}$ (f.t. candidate's values for a, b, c) m1

$x = \frac{21}{2}, -\frac{9}{2}$ (both values) A1

5. (*a*) An expression for $b^2 - 4ac$, with at least two of a, b or c correct M1

$b^2 - 4ac = (2k - 5)^2 - 4 \times k \times (k - 6)$ A1

Putting $b^2 - 4ac < 0$ m1

$k < -\frac{25}{4}$ (or equivalent) A1

(*b*) $k = -\frac{25}{4}$ [f.t. the end point(s) of the candidate's range in (*a*)] B1

6. (*a*) $\left(1 - \frac{x}{2}\right)^8 = 1 - 4x + 7x^2 - 7x^3 + \ldots$ B1 B1 B1 B1

(– 1 for further incorrect simplification)

(*b*) First term = 2^n B1

$2^n = 32 \Rightarrow n = 5$ B1

Second term = $n \times 2^{n-1} \times ax$ B1

$a = -3$ (f.t. candidate's value for n) B1

7. (*a*) $y + \delta y = 9(x + \delta x)^2 - 8(x + \delta x) - 3$ B1

Subtracting y from above to find δy M1

$\delta y = 18x\delta x + 9(\delta x)^2 - 8\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 18x - 8$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = 3 \times (-6) \times x^{-7} - 4 \times \frac{5}{3} \times x^{2/3}$ B1 B1

8. (*a*) Use of $f(3) = 0$ M1

$27p - 117 - 57 + 12 = 0 \Rightarrow p = 6$ (convincing) A1

Special case

Candidates who assume $p = 6$ and show $f(3) = 0$ are awarded B1

(*b*) $f(x) = (x - 3)(6x^2 + ax + b)$ with one of a, b correct M1

$f(x) = (x - 3)(6x^2 + 5x - 4)$ A1

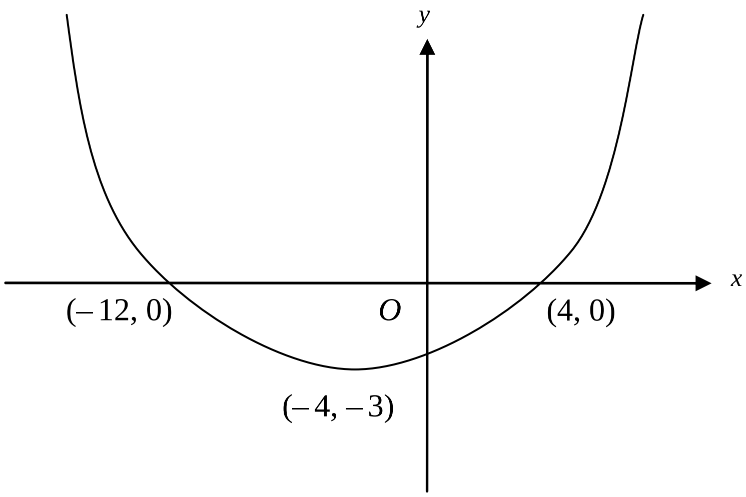
$f(x) = (x - 3)(2x - 1)(3x + 4)$ (f.t. only $6x^2 - 5x - 4$ in above line) A1

Roots are $x = 3, \frac{1}{2}, -\frac{4}{3}$ (f.t. for factors $2x \pm 1$, $3x \pm 4$) A1

Special case

Candidates who find one of the remaining factors, $(2x - 1)$ or $(3x + 4)$, using e.g. factor theorem, are awarded B1

9. (*a*)



Concave up curve and y-coordinate of minimum = – 3	B1
x-coordinate of minimum = – 4	B1
Both points of intersection with x-axis	B1

(*b*) **Either:**

Any graph of the form $y = af(x)$ (with $a \neq 0$) will intersect the x-axis at (– 6, 0) and (2, 0) and thus not pass through the origin.

Or:

$f(0) \neq 0$ and since $a \neq 0$, $af(0) \neq 0$. Thus any graph of the form $y = af(x)$ will not pass through the origin. E1

10. (*a*)

$L = x + 2y$

$800 = xy$ (both equations) M1

$L = x + \frac{1600}{x}$ (convincing) A1

(*b*)

$\frac{dL}{dx} = 1 + 1600 \times (-1) \times x^{-2}$ B1

Putting derived $\frac{dL}{dx} = 0$ M1

$x = 40, (-40)$ (f.t. candidate's $\frac{dL}{dx}$) A1

Stationary value of L at $x = 40$ is 80 (c.a.o) A1

A correct method for finding nature of the stationary point yielding a minimum value (for $x > 0$) B1

C2

1.

1	0·1111111111		
1·5	0·1709352011		
2	0·2329431339		
2·5	0·2969522777		
3	0·3628469322	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}5$ M1

$$I \approx \frac{0{\cdot}5}{2} \times \{0{\cdot}1111111111 + 0{\cdot}3628469322 + 2(0{\cdot}1709352011 + 0{\cdot}2329431339 + 0{\cdot}2969522777)\}$$

$I \approx 1{\cdot}875619269 \times 0{\cdot}5 \div 2$

$I \approx 0{\cdot}4689048172$

$I \approx 0{\cdot}4689$ (f.t. one slip) A1

Special case for candidates who put $h = 0{\cdot}4$

1	0·1111111111		
1·4	0·1587880562		
1·8	0·2078915826		
2·2	0·2583141854		
2·6	0·3099833063		
3	0·3628469322	(all values correct)	B1

Correct formula with $h = 0{\cdot}4$ M1

$$I \approx \frac{0{\cdot}4}{2} \times \{0{\cdot}1111111111 + 0{\cdot}3628469322 + 2(0{\cdot}1587880562 + 0{\cdot}2078915826 + 0{\cdot}2583141854 + 0{\cdot}3099833063)\}$$

$I \approx 2{\cdot}343912304 \times 0{\cdot}4 \div 2$

$I \approx 0{\cdot}4687824609$

$I \approx 0{\cdot}4688$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $4(1 - \sin^2\theta) - 2\sin^2\theta - \sin\theta + 8 = 0$,

(correct use of $\cos^2\theta = 1 - \sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$6\sin^2\theta + \sin\theta - 12 = 0 \Rightarrow (2\sin\theta + 3)(3\sin\theta - 4) = 0$

$\Rightarrow \sin\theta = -\frac{3}{2},\ \sin\theta = \frac{4}{3}$ (c.a.o.) A1

$-1 \le \sin\theta \le 1 \Rightarrow$ no such θ can exist

(f.t. only if candidate has 2 real values for $\sin\theta$, **neither** of which satisfies $-1 \le \sin\theta \le 1$) E1

(*b*) $2x - 75° = -31°, 211°, 329°$, (one value) B1

$x = 22°, 143°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.

(*c*) $4\sin\phi + 7\sin\phi\cos\phi = 0$ **or** $4\tan\phi + 7\tan\phi\cos\phi = 0$

or $\sin\phi\left(\frac{4}{\cos\phi} + 7\right) = 0$ M1

$\sin\phi = 0$ (or $\tan\phi = 0$), $\cos\phi = -\frac{4}{7}$ (both values) A1

$\phi = 0°, 180°$ (both values) A1

$\phi = 124{\cdot}85°$ (c.a.o.) A1

Note: Subtract a maximum of 1 mark for each additional root in range for each branch, ignore roots outside range.

Special Case:

No factorisation but division throughout by $\sin\phi$ (or $\tan\phi$) to yield

$4 + 7\cos\phi = 0$ (or equivalent) M1

$\phi = 124{\cdot}85°$ A1

3. (*a*) $\frac{\sin ACB}{19} = \frac{\sin 25°}{12}$

(substituting the correct values in the correct places in the sin rule) M1

$ACB = 42°, 138°$ (both values) A1

(*b*) (i) $B\hat{A}C + 25° + 138° = 180°$

(f.t. either of candidate's values for ACB) M1

$B\hat{A}C = 17°$ (f.t. candidate's obtuse value for ACB) A1

(ii) Area of triangle $ABC = \frac{1}{2} \times 19 \times 12 \times \sin 17°$

(substituting 19, 12 and candidate's derived value for $B\hat{A}C$ in the correct places in the area formula) M1

Area of triangle $ABC = 33{\cdot}33\ \text{cm}^2$. (c.a.o.) A1

4. (*a*) (i) nth term $= 4 + 6(n - 1) = 4 + 6n - 6 = 6n - 2$ (convincing) B1

(ii) $S_n = 4 + 10 + \ldots + (6n - 8) + (6n - 2)$

$S_n = (6n - 2) + (6n - 8) + \ldots + 10 + 4$

Reversing and adding M1

Either:

$2S_n = (6n + 2) + (6n + 2) + \ldots + (6n + 2) + (6n + 2)$

Or:

$2S_n = (6n + 2) + \ldots$ (n times) A1

$2S_n = n(6n + 2)$

$S_n = n(3n + 1)$ (convincing) A1

(*b*) (i) $a + 9d = 4 \times (a + 4d)$ B1

$3a + 7d = 0$

$\frac{15}{2} \times (2a + 14d) = 210$ B1

$a + 7d = 14$

An attempt to solve the candidate's two derived linear equations simultaneously by eliminating one unknown M1

$d = 3, a = -7$ (c.a.o.) A1

(ii) $-7 + (k - 1) \times 3 = 200$

(f.t. candidate's derived values for a and d) M1

$k = 70$ (c.a.o.) A1

5. (*a*) $r = \frac{2304}{576} = 4$ (c.a.o.) B1

$t_5 = \frac{576}{4^3}$ (f.t. candidate's value for r) M1

$t_5 = 9$ (c.a.o.) A1

(*b*) (i) $ar^2 = 24$ B1

$ar + ar^2 + ar^3 = -56$ B1

An attempt to solve the candidate's equations simultaneously by eliminating a M1

$\frac{r^2}{r + r^2 + r^3} = -\frac{24}{56} \Rightarrow 3r^2 + 10r + 3 = 0$ (convincing) A1

(ii) $r = -\frac{1}{3}$ ($r = -3$ discarded, c.a.o.) B1

$a = 216$

(f.t. candidate's derived value for r, provided $|r| < 1$) B1

$S_\infty = \frac{216}{1 - (-^1/_3)}$ (use of formula for sum to infinity)

(f.t. candidate's derived values for r and a) M1

$S_\infty = 162$ (f.t. candidate's derived values for r and a) A1

6. (*a*) $3 \times \frac{x^{1/2}}{1/2} - 6 \times \frac{x^{7/3}}{7/3} + c$ B1, B1
(–1 if no constant term present)

(*b*) (i) $6 + 5x - x^2 = 4x$ M1
An attempt to rewrite and solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b$ = candidate's constant m1
$(x + 2)(x - 3) = 0 \Rightarrow x = 3$ (c.a.o.) A1

(ii) Use of integration to find the area under the curve M1
$\int 6\,dx = 6x$, $\int 5x\,dx = \frac{5x^2}{2}$, $\int x^2\,dx = (1/3)x^3$,
(correct integration) B1
Correct method of substitution of candidate's limits m1

$\left[6x + (5/2)x^2 - (1/3)x^3\right]_{-1}^{3}$

$= (18 + 45/2 - 9) - (-6 + 5/2 - (-1/3)) = 104/3$

Use of a correct method to find the area of the triangle
(f.t. candidate's coordinates for A) M1
Use of – 1 and candidate's value for x_A as limits and trying to find total area by subtracting area of triangle from area under curve m1
Shaded area = 104/3 – 18 = 50/3 (c.a.o.) A1

7. (*a*) Let $p = \log_a x$, $q = \log_a y$
Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1
$\frac{x}{y} = \frac{a^p}{a^q} = a^{p-q}$ (the laws of indicies) B1
$\log_a x/y = p - q$ (the relationship between log and power)
$\log_a x/y = p - q = \log_a x - \log_a y$ (convincing) B1

(*b*) $\log_a(6x^2 + 9x + 2) - \log_a x = \log_a \left(\frac{6x^2 + 9x + 2}{x}\right)$
(subtraction law) B1
$4 \log_a 2 = \log_a 2^4$ (power law) B1
$\frac{6x^2 + 9x + 2}{x} = 2^4$ (removing logs) M1
An attempt to solve quadratic equation with three terms in x, either by using the quadratic formula or by getting the expression into the form $(ax + b)(cx + d)$, with $a \times c$ = candidate's coefficient of x^2 and $b \times d$ = candidate's constant m1
$6x^2 - 7x + 2 = 0 \Rightarrow (2x - 1)(3x - 2) = 0 \Rightarrow x = {}^1/_2, {}^2/_3$
(both values, c.a.o.) A1
Note: Answer only with no working earns 0 marks

8. (*a*) (i) $A(3, -1)$ B1

(ii) A correct method for finding radius M1

Radius = $\sqrt{29}$ (convincing) A1

(*b*) **Either:**

$RQ = \sqrt{18}$ or $RP = \sqrt{98}$ (o.e.) B1

Correct substitution of candidate's values in an expression for sin Q, cos Q or tan Q M1

$PQR = 66{\cdot}8°$ (c.a.o) A1

Or:

$RQ = \sqrt{18}$ or $RP = \sqrt{98}$ B1

Correct substitution of candidate's values in the cos rule to find cos Q M1

$PQR = 66{\cdot}8°$ (c.a.o) A1

(*c*) $AT^2 = 65$ (f.t. candidate's coordinates for A) B1

Use of $ST^2 = AT^2 - AS^2$ with candidate's derived value for AT M1

$ST = 6$ (f.t. one slip) A1

9. Area of sector $AOB = {}^1/_2 \times r^2 \times 2{\cdot}6$ B1

Area of triangle $AOB = {}^1/_2 \times r^2 \times \sin 2{\cdot}6$ B1

Area of minor segment = ${}^1/_2 \times r^2 \times 2{\cdot}6 - {}^1/_2 \times r^2 \times \sin 2{\cdot}6 = 1{\cdot}0422r^2$ B1

Use of a valid method for finding the area of the major segment M1

Area of major segment = $2{\cdot}099r^2$

$\Rightarrow$ area of major segment $\approx 2 \times$ area of minor segment (convincing) A1

C3

1. (*a*)

0	0
$\pi/9$	$-0{\cdot}062202456$
$2\pi/9$	$-0{\cdot}266515091$
$\pi/3$	$-0{\cdot}693147181$
$4\pi/9$	$-1{\cdot}750723994$

(5 values correct) B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = \pi/9$ M1

$$I \approx \frac{\pi/9}{3} \times \{0 + (-1{\cdot}750723994) + 4[(-0{\cdot}062202456) + (-0{\cdot}693147181)] + 2(-0{\cdot}266515091)\}$$

$I \approx -5{\cdot}305152724 \times (\pi/9) \div 3$

$I \approx -0{\cdot}617282549$

$I \approx -0{\cdot}6173$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$$\int_0^{4\pi/9} \ln(\sec x)\,dx \approx 0{\cdot}6173$$ (f.t. candidate's answer to (*a*)) B1

2. (*a*) $7\,\text{cosec}^2\theta - 4(\text{cosec}^2\theta - 1) = 16 + 5\,\text{cosec}\,\theta$

(correct use of $\cot^2\theta = \text{cosec}^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in cosec θ, either by using the quadratic formula or by getting the expression into the form $(a\,\text{cosec}\,\theta + b)(c\,\text{cosec}\,\theta + d)$, with $a \times c$ = candidate's coefficient of $\text{cosec}^2\theta$ and $b \times d$ = candidate's constant m1

$3\,\text{cosec}^2\theta - 5\,\text{cosec}\,\theta - 12 = 0 \Rightarrow (\text{cosec}\,\theta - 3)(3\,\text{cosec}\,\theta + 4) = 0$

$\Rightarrow \text{cosec}\,\theta = 3,\ \text{cosec}\,\theta = -\frac{4}{3}$

$\Rightarrow \sin\theta = \frac{1}{3},\ \sin\theta = -\frac{3}{4}$ (c.a.o.) A1

$\theta = 19{\cdot}47°, 160{\cdot}53°$ B1

$\theta = 311{\cdot}41°, 228{\cdot}59°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks

$\sin\theta = +, +$, f.t. for 1 mark

(*b*) $\sec\phi \ge 1$, $\text{cosec}\,\phi \ge 1$ and thus $4\sec\phi + 3\,\text{cosec}\,\phi \ge 7$ E1

3. (*a*) $\frac{d}{dx}(x^3) = 3x^2$ $\frac{d}{dx}(1) = 0$ $\frac{d}{dx}(\pi^2/4) = 0$ B1

$\frac{d}{dx}(2x\cos y) = 2x(-\sin y)\frac{dy}{dx} + 2\cos y$ B1

$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{3}{2 - \pi}$ (c.a.o.) B1

(*b*) $\frac{d^2y}{dx^2} = \frac{d}{dx}(x^2y) = x^2\frac{dy}{dx} + 2xy$ B1

Substituting x^2y for $\frac{dy}{dx}$ in candidate's derived expression for $\frac{d^2y}{dx^2}$ M1

$\frac{d^2y}{dx^2} = x^2(x^2y) + 2xy = x^4y + 2xy$ (o.e.) (c.a.o.) A1

4. (*a*)

candidate's x-derivative = $\dfrac{1}{1+t^2}$ B1

candidate's y-derivative = $\dfrac{1}{t}$ B1

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = \dfrac{1+t^2}{t}$ A1

(*b*)

$\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) = -t^{-2} + 1$ (o.e.) B1

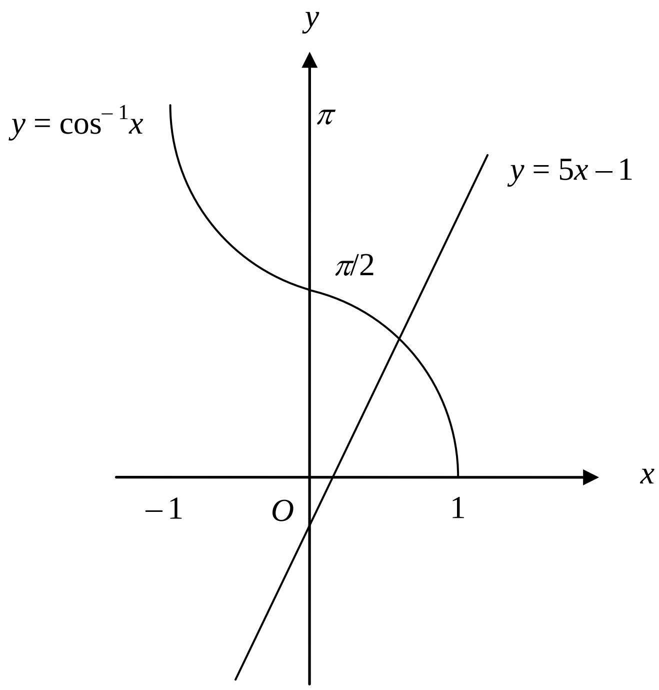
Use of $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \div$ candidate's x-derivative M1

$\dfrac{d^2y}{dx^2} = (-t^{-2} + 1)(1 + t^2)$ (o.e.) (f.t. one slip) A1

$\dfrac{d^2y}{dx^2} = 0 \Rightarrow t = 1$ (c.a.o.) A1

$\dfrac{d^2y}{dx^2} = 0 \Rightarrow x = \dfrac{\pi}{4}$ (f.t. candidate's derived value for t) A1

5. (*a*)



Correct shape for $y = \cos^{-1}x$ B1

A straight line with negative y-intercept and positive gradient intersecting once with $y = \cos^{-1}x$ in the first quadrant. B1

(*b*) $x_0 = 0{\cdot}4$

$x_1 = 0{\cdot}431855896$ (x_1 correct, at least 4 places after the point) B1

$x_2 = 0{\cdot}424849379$

$x_3 = 0{\cdot}426400166$

$x_4 = 0{\cdot}426057413 = 0{\cdot}4261$ (x_4 correct to 4 decimal places) B1

Let $h(x) = \cos^{-1}x - 5x + 1$

An attempt to check values or signs of $h(x)$ at $x = 0{\cdot}42605$, $x = 0{\cdot}42615$ M1

$h(0{\cdot}42605) = 4{\cdot}24 \times 10^{-4} > 0$, $h(0{\cdot}42615) = -1{\cdot}86 \times 10^{-4} < 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}4261$ correct to four decimal places A1

6. (*a*) (i) $\frac{dy}{dx} = \frac{a + bx}{4x^2 - 3x - 5}$ (including $a = 1$, $b = 0$) M1

$\frac{dy}{dx} = \frac{8x - 3}{4x^2 - 3x - 5}$ A1

(ii) $\frac{dy}{dx} = e^{\sqrt{x}} \times f(x)$ ($f(x) \neq 1, 0$) M1

$\frac{dy}{dx} = e^{\sqrt{x}} \times \frac{1}{2} x^{-1/2}$ A1

(iii) $\frac{dy}{dx} = \frac{(a - b\sin x) \times m\cos x - (a + b\sin x) \times k\cos x}{(a - b\sin x)^2}$

($m = \pm b$, $k = \pm b$) M1

$\frac{dy}{dx} = \frac{(a - b\sin x) \times b\cos x - (a + b\sin x) \times (-b)\cos x}{(a - b\sin x)^2}$ A1

$\frac{dy}{dx} = \frac{2ab\cos x}{(a - b\sin x)^2}$ A1

(*b*) $\frac{d}{dx}(\cot x) = \frac{d}{dx}(\tan x)^{-1} = (-1) \times (\tan x)^{-2} \times f(x)$ ($f(x) \neq 1, 0$) M1

$\frac{d}{dx}(\tan x)^{-1} = (-1) \times (\tan x)^{-2} \times \sec^2 x$ A1

$\frac{d}{dx}(\tan x)^{-1} = -\text{cosec}^2 x$ (convincing) A1

7. (*a*) (i) $\int \frac{(7x^2 - 2)}{x}\, dx = \int 7x\, dx - \int \frac{2}{x}\, dx$

Correctly rewriting as two terms and an attempt to integrate M1

$\int \frac{(7x^2 - 2)}{x}\, dx = \frac{7}{2}x^2 - 2 \ln x + c$ A1 A1

(ii) $\int \sin(^{2x}/_3 - \pi)\, dx = k \times \cos(^{2x}/_3 - \pi) + c$

$(k = -1, -^3/_2, ^3/_2, -^2/_3)$ M1

$\int \sin(^{2x}/_3 - \pi)\, dx = -^3/_2 \times \cos(^{2x}/_3 - \pi) + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int (5x - 14)^{-1/4}\, dx = \frac{k \times (5x - 14)^{3/4}}{3/4}$ $(k = 1, 5, ^1/_5)$ M1

$\int (5x - 14)^{-1/4}\, dx = {}^1/_5 \times \frac{(5x - 14)^{3/4}}{3/4}$ A1

A correct method for substitution of the correct limits limits in an expression of the form $m \times (5x - 14)^{3/4}$ M1

$\int_3^6 (5x - 14)^{-1/4}\, dx = \frac{28}{15}$ (= 1·867)

(f.t. only for solutions of $\frac{28}{3}$ (= 9·333) and $\frac{140}{3}$ (= 46·667) from $k = 1$, $k = 5$ respectively) A1

Note: Answer only with no working shown earns 0 marks

8. (*a*) Trying to solve either $3x - 5 \le 1$ or $3x - 5 \ge -1$ M1

$3x - 5 \le 1 \Rightarrow x \le 2$

$3x - 5 \ge -1 \Rightarrow x \ge {}^4/_3$ (both inequalities) A1

Required range: ${}^4/_3 \le x \le 2$ (f.t. one slip) A1

Alternative mark scheme

$(3x - 5)^2 \le 1$

(squaring both sides, forming and trying to solve quadratic) M1

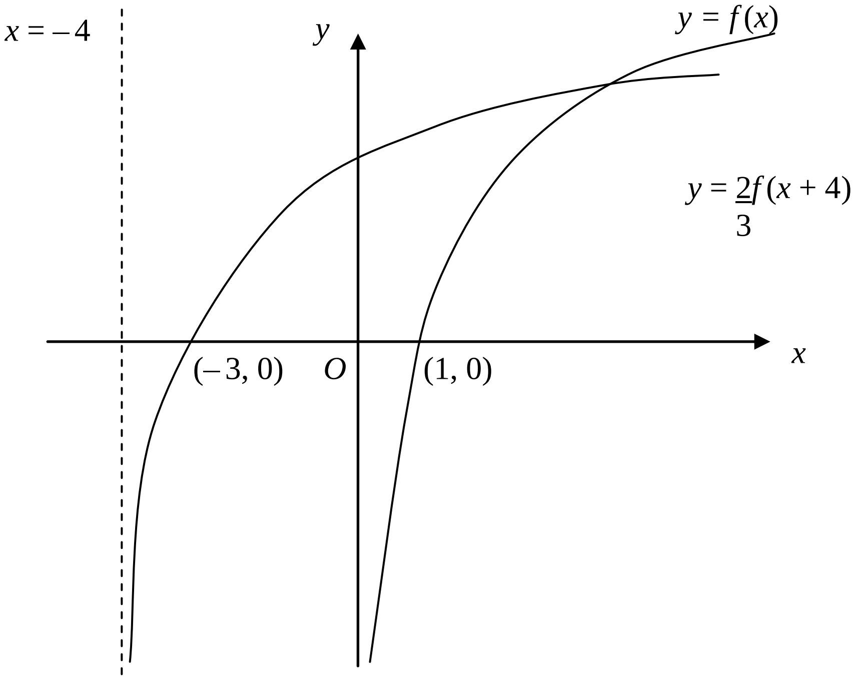
Critical values $x = {}^4/_3$ and $x = 2$ A1

Required range: ${}^4/_3 \le x \le 2$ (f.t. one slip in critical values) A1

(*b*) ${}^4/_3 \le 1/y \le 2$ (f.t. candidate's $a \le x \le b$, $a > 0$, $b > 0$) M1

${}^1/_2 \le y \le {}^3/_4$ (f.t. candidate's $a \le x \le b$, $a > 0$, $b > 0$) A1

9.



Correct shape, including the fact that the y-axis is an asymptote for $y = f(x)$ at $-\infty$ B1

$y = f(x)$ cuts x-axis at (1, 0) B1

Correct shape, including the fact that $x = -4$ is an asymptote for $y = \frac{2}{3}f(x + 4)$ at $-\infty$ B1

$y = \frac{2}{3}f(x + 4)$ cuts x-axis at $(-3, 0)$ (f.t. candidate's x-intercept for $f(x)$) B1

The diagram shows that the graph of $y = f(x)$ is steeper than the graph of $y = \frac{2}{3}f(x + 4)$ in the first quadrant B1

10. (*a*) Choice of h, k such that $h(x) = k(x) + c$, $c \neq 0$ M1

Convincing verification of the fact that $h'(x) = k'(x)$ A1

(*b*) (i) $y - 3 = 2\ln(4x + 5)$ B1

An attempt to express candidate's equation as an exponential equation M1

$x = \frac{(e^{(y-3)/2} - 5)}{4}$ (c.a.o.) A1

$f^{-1}(x) = \frac{(e^{(x-3)/2} - 5)}{4}$

(f.t. one slip in candidate's expression for x) A1

(ii) $D(f^{-1}) = [10, 14]$ B1 B1

(iii) $gf(x) = e^{2\ln(4x+5)+3}$ B1

$e^{2\ln(4x+5)} = (4x + 5)^2$ B1

$gf(x) = e^3(4x + 5)^2$ (c.a.o.) B1

1. (*a*) $f(x) \equiv \dfrac{A}{(x+3)^2} + \dfrac{B}{(x+3)} + \dfrac{C}{(x-1)}$ (correct form) M1

$2x^2 + 5x + 25 \equiv A(x-1) + B(x+3)(x-1) + C(x+3)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -7,\ C = 2,\ B = 0$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) $\displaystyle\int f(x)\,dx = \frac{7}{(x+3)} + 2\ln(x-1)$ B1 B1

(f.t. candidate's values for *A*, *B*, *C*)

$\displaystyle\int_3^{10} f(x)\,dx = \left[\frac{7}{13} + 2\ln 9\right] - \left[\frac{7}{6} + 2\ln 2\right] = 2{\cdot}38$ (c.a.o.) B1

Note: Answer only with no working earns 0 marks

2. (*a*) $4x^3 + 3x^2\dfrac{dy}{dx} + 6xy - 4y\dfrac{dy}{dx} = 0$

$\left[3x^2\dfrac{dy}{dx} + 6xy\right]$ B1

$\left[4x^3 - 4y\dfrac{dy}{dx}\right]$ B1

$\dfrac{dy}{dx} = \dfrac{4x^3 + 6xy}{4y - 3x^2}$ (convincing) B1

(*b*) $4y - 3x^2 = 0$ M1

Either: Substituting $\dfrac{3x^2}{4}$ for *y* in the equation of *C* and an attempt to collect terms m1

$x^4 = 16 \Rightarrow x = (\pm)\,2$ A1

$y = 3$ (for both values of *x*)

(f.t. $x^4 = a$, $a \neq 16$, provided both *x* values are checked) A1

Or: Substituting $\dfrac{4y}{3}$ for x^2 in the equation of *C* and an attempt to collect terms m1

$y^2 = 9 \Rightarrow y = (\pm)\,3$ A1

$y = 3 \Rightarrow x = \pm\,2$ (f.t. $y^2 = b$, $b \neq 9$) A1

3. (*a*) $\frac{\tan x + \tan 45°}{1 - \tan x \tan 45°} = 8 \tan x$ (correct use of formula for $\tan(x + 45°)$) M1

Use of $\tan 45° = 1$ and an attempt to form a quadratic in $\tan x$ by cross multiplying and collecting terms M1

$8\tan^2 x - 7\tan x + 1 = 0$ (c.a.o.) A1

Use of a correct method to solve the candidate's derived quadratic in $\tan x$ m1

$x = 34{\cdot}8°, 10{\cdot}2°$ (both values)

(f.t. one slip in candidate's derived quadratic in $\tan x$ provided all three method marks have been awarded) A1

(*b*) (i) $R = 7$ B1

Correctly expanding $\sin(\theta - \alpha)$, correctly comparing coefficients and using either $7\cos\alpha = \sqrt{13}$ **or** $7\sin\alpha = 6$ **or** $\tan\alpha = \frac{6}{\sqrt{13}}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 59°$ (c.a.o) A1

(ii) $\sin(\theta - \alpha) = -\frac{4}{7}$

(f.t. candidate's values for R, α) B1

$\theta - 59° = -34{\cdot}85°, 214{\cdot}85°, 325{\cdot}15°,$

(at least one value, f.t. candidate's values for R, α) B1

$\theta = 24{\cdot}15°, 273{\cdot}85°$ (c.a.o.) B1

4. (*a*)

$V = \pi \int_0^a (mx)^2 \, dx$ M1

$\int (mx)^2 \, dx = \frac{m^2 x^3}{3}$ B1

$V = \pi \frac{m^2 a^3}{3}$ (c.a.o.) A1

(*b*) (i) Substituting $\frac{b}{a}$ for m in candidate's derived expression for V M1

$V = \pi \frac{b^2 a}{3}$ (c.a.o.) A1

(ii) This is the volume of a cone of (vertical) height a and (base) radius b E1

5. $\left(1 + \frac{x}{8}\right)^{-1/2} = 1 - \frac{x}{16} + \frac{3x^2}{512}$ — $\left(1 - \frac{x}{16}\right)$ B1; $\left(\frac{3x^2}{512}\right)$ B1

$|x| < 8$ or $-8 < x < 8$ B1

$\frac{2}{3}\sqrt{2} \approx 1 - \frac{1}{16} + \frac{3}{512}$ (f.t. candidate's coefficients) B1

Either: $\sqrt{2} \approx \frac{1449}{1024}$ (c.a.o.)

Or: $\sqrt{2} \approx \frac{2048}{1449}$ (c.a.o.) B1

6. (*a*) (i) candidate's *x*-derivative = $2at$

candidate's *y*-derivative = $2a$ (at least one term correct)

and use of

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

Gradient of tangent at $P = \frac{1}{p}$ (c.a.o.) A1

(ii) Equation of tangent at *P*: $y - 2ap = \frac{1}{p}(x - ap^2)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

Equation of tangent at *P*: $py = x + ap^2$ A1

(*b*) (i) Gradient $PQ = \frac{2ap - 2aq}{ap^2 - aq^2}$ B1

Use of $ap^2 - aq^2 = a(p + q)(p - q)$ B1

Gradient $PQ = \frac{2}{p + q}$ (c.a.o.) B1

(ii) As the point *Q* approaches *P*, *PQ* becomes a tangent

$\lim_{q \to p}$ (gradient PQ) $= \frac{2}{2p} = \frac{1}{p}$. E1

7. (*a*) $\displaystyle\int \frac{x^2}{(12 - x^3)^2}\,dx = \int \frac{k}{u^2}\,du$ ($k = {}^1/_3$, $-{}^1/_3$, 3 or -3) M1

$\displaystyle\int \frac{a}{u^2}\,du = a \times \frac{u^{-1}}{-1}$ B1

Either: Correctly inserting limits of 12, 4 in candidate's bu^{-1}
or: Correctly inserting limits of 0, 2 in candidate's $b(12 - x^3)^{-1}$
M1

$\displaystyle\int_0^2 \frac{x^2}{(12 - x^3)^2}\,dx = \frac{1}{18}$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

(*b*) (i) $u = x \Rightarrow du = dx$ (o.e.) B1

$dv = \cos 2x\,dx \Rightarrow v = \frac{1}{2}\sin 2x$ (o.e.) B1

$\displaystyle\int x\cos 2x\,dx = x \times \frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x \times dx$ M1

$\displaystyle\int x\cos 2x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$ (c.a.o.) A1

(ii) $\displaystyle\int x\sin^2 x\,dx = \int x\left[\frac{k}{2} - \frac{m}{2}\cos 2x\right]dx$ (o.e.)

($k = \mathbf{1}, -1$, $m = \mathbf{1}, -1$) M1

$\displaystyle\int x\sin^2 x\,dx = \frac{1}{2}\int x\,dx - \frac{1}{2}\int x\cos 2x\,dx$ A1

$\displaystyle\int x\sin^2 x\,dx = \frac{x^2}{4} - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + c$

(f.t. only candidate's answer to (*b*)(*i*)) A1

8. (*a*) (i) $\mathbf{AB} = -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$ B1

(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of AB M1

$\mathbf{r} = 5\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k})$ (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*b*) $5 - \lambda = 2 + \mu$
$-1 - 2\lambda = -3 + \mu$
$-1 + 7\lambda = -4 - \mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her equation of AB)

$\lambda = -1$, $\mu = 4$ (o.e.) (c.a.o.) A1

Correct verification that values of λ and μ satisfy third equation A1

Position vector of point of intersection is $6\mathbf{i} + \mathbf{j} - 8\mathbf{k}$

(f.t. one slip) A1

9. (*a*) $\frac{dP}{dt} = kP^2$ B1

(*b*) $\int \frac{dP}{P^2} = \int k\, dt$ M1

$-\frac{1}{P} = kt + c$ (o.e.) A1

$c = -\frac{1}{A}$ (c.a.o.) A1

$-\frac{1}{P} = kt - \frac{1}{A} \Rightarrow kt = \frac{1}{A} - \frac{1}{P} \Rightarrow \frac{1}{k}\left(\frac{P-A}{PA}\right) = t$ (convincing) A1

(*c*) $\frac{1}{k}\left(\frac{800-A}{800A}\right) = 3,\quad \frac{1}{k}\left(\frac{900-A}{900A}\right) = 4$ (both equations) B1

An attempt to solve these equations simultaneously by eliminating *k* M1

$A = 600$ (c.a.o.) A1

10. Assume that 4 is a factor of $a + b$.
Then there exists an integer c such that $a + b = 4c$.
Similarly, there exists an integer d such that $a - b = 4d$. B1
Adding, we have $2a = 4c + 4d$. B1
Therefore $a = 2c + 2d$, an even number, which contradicts the fact that a is odd. B1

Ques	Solution	Mark	Notes
1	$f(x+h)-f(x)=\dfrac{1}{(x+h)^2-(x+h)}-\dfrac{1}{x^2-x}$	**M1A1**	
	$=\dfrac{x^2-x-[(x+h)^2-(x+h)]}{[(x+h)^2-(x+h)](x^2-x)}$	**A1**	
	$=\dfrac{x^2-x-[x^2+2hx+h^2-x-h)]}{[(x+h)^2-(x+h)](x^2-x)}$	**A1**	
	$=\dfrac{-2hx-h^2+h}{[(x+h)^2-(x+h)](x^2-x)}$	**A1**	
	$f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$	**M1**	
	$=\lim_{h\to 0}\dfrac{-2x-h+1}{[(x+h)^2-(x+h)](x^2-x)}=\dfrac{-2x+1}{(x^2-x)^2}$	**A1**	oe
2(a)	The reflection matrix for $y = x$ is $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	**B1**	Allow the use of 3×3 matrices
	The reflection matrix for $y = -x$ is $\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$	**B1**	
	It follows that $\mathbf{T}=\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	**M1**	Special case B1 for matrices the wrong way round
	$=\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$ cao	**A1**	Do not award this A1 for a 3×3 matrix
(b)	T therefore corresponds to a rotation through 180° about the origin. cao	**B1**	
3(a)	$\dfrac{2+\mathrm{i}}{1-\mathrm{i}}=\dfrac{(2+\mathrm{i})(1+\mathrm{i})}{(1-\mathrm{i})(1+\mathrm{i})}$	**M1**	
	$=\dfrac{2+3\mathrm{i}+\mathrm{i}^2}{1-\mathrm{i}+\mathrm{i}-\mathrm{i}^2}$	**A1**	
	$=\dfrac{1}{2}+\dfrac{3}{2}\mathrm{i}$	**A1**	
	Let $z = x + \mathrm{i}y$ so that $\bar{z}=x-\mathrm{i}y$		
	$2(x+\mathrm{i}y)-\mathrm{i}(x-\mathrm{i}y)=\dfrac{1}{2}+\dfrac{3}{2}\mathrm{i}$	**M1**	FT their above result
	$2x-y=\dfrac{1}{2}$; $2y-x=\dfrac{3}{2}$	**A1**	
	$x=\dfrac{5}{6}$; $y=\dfrac{7}{6}$ $\left(\text{so } z=\dfrac{5}{6}+\dfrac{7}{6}\mathrm{i}\right)$	**A1**	

(b)	Mod = $\sqrt{(-20)^2+(-21)^2}=29$ $\tan^{-1}\left(\frac{21}{20}\right)=0.81$ or $46.4°$ si Arg = 0.81 + π = 3.95 or 46.4° + 180° = 226.4°	**B1** **B1** **B1**	 Accept – 2.33 or – 133.6°
4(a) **(b)(i)** **(ii)**	$\det(\mathbf{M}) = 1(10-1)+2(1-4)+1(2-5)$ $= 0$ **M** is therefore singular. Using row operations, $\begin{bmatrix}1&2&1\\0&1&-1\\0&1&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-2\\2-\mu\end{bmatrix}$ It follows that $-2 = 2-\mu$ so $\mu = 4$ Let $z = \alpha$. Then $y = \alpha - 2$. and $x = 6 - 3\alpha$.	**M1** **A1** **A1** **M1** **A1** **A1** **A1** **M1** **A1** **A1**	
5	Let the roots be a, ar, ar^2. Then, $a+ar+ar^2=4$ $a^2r+a^2r^2+a^2r^3=-8$ Dividing, $ar=-2$ $k=-a^3r^3=8$	**M1** **A1** **M1** **A1** **A1**	Allow *a*/*r*, *a*, *ar*
6(a) **(b)** **(c)**	$\begin{bmatrix}3&2&4\\3&3&6\\2&2&3\end{bmatrix}\begin{bmatrix}3&-2&0\\-3&-1&6\\0&2&-3\end{bmatrix}=\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$ It follows that $\boldsymbol{A}^{-1}=\frac{1}{3}\boldsymbol{B}\quad\left(=\frac{1}{3}\begin{bmatrix}3&-2&0\\-3&-1&6\\0&2&-3\end{bmatrix}\right)$ $\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{3}\begin{bmatrix}3&-2&0\\-3&-1&6\\0&2&-3\end{bmatrix}\begin{bmatrix}14\\18\\11\end{bmatrix}=\begin{bmatrix}2\\2\\1\end{bmatrix}$	**B2** **M1A1** **M1A1**	Award B1 if 1 error, B0 more than 1 error M1A0 for 3**B** FT their $\boldsymbol{A}^{-1}$

7(a)	Let $\dfrac{2}{n(n+2)}=\dfrac{A}{n}+\dfrac{B}{n+2}=\dfrac{A(n+2)+Bn}{n(n+2)}$ $A=1;B=-1$ $\left(\dfrac{2}{n(n+2)}=\dfrac{1}{n}-\dfrac{1}{(n+2)}\right)$	**M1** **A1A1**	
(b)	$S_n=1\quad -\dfrac{1}{3}$ $\dfrac{1}{2}\quad -\dfrac{1}{4}$ $\dfrac{1}{3}\quad -\dfrac{1}{5}$ .... $\dfrac{1}{(n-1)}\quad -\dfrac{1}{(n+1)}$ $\dfrac{1}{n}\quad -\dfrac{1}{(n+2)}$ $=1+\dfrac{1}{2}-\dfrac{1}{(n+1)}-\dfrac{1}{(n+2)}$ $=\dfrac{3(n+1)(n+2)-2(n+2)-2(n+1)}{2(n+1)(n+2)}$ $=\dfrac{3n^2+5n}{2(n+1)(n+2)}$	**M1** **A1** **A1** **A1** **A1**	
8(a)	$\mathbf{A}^2=\begin{bmatrix}1&0\\2&1\end{bmatrix}\begin{bmatrix}1&0\\2&1\end{bmatrix}=\begin{bmatrix}1&0\\4&1\end{bmatrix}$ $2\mathbf{A}-\mathbf{I}=2\begin{bmatrix}1&0\\2&1\end{bmatrix}-\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}1&0\\4&1\end{bmatrix}$ Hence equal.	**B1** **B1**	
(b)	METHOD 1 Let the result be true for $n=k$, ie $\mathbf{A}^k=k\mathbf{A}-(k-1)\mathbf{I}$ Consider, for $n=k+1$, $\mathbf{A}^{k+1}=k\mathbf{A}^2-(k-1)\mathbf{A}$ $=k(2\mathbf{A}-\mathbf{I})-(k-1)\mathbf{A}$ $=(k+1)\mathbf{A}-k\mathbf{I}$ Hence true for $n=k\Rightarrow$ true for $n=k+1$ and since trivially true for $n=1$ ($\mathbf{A}=\mathbf{A}$), the result is proved by induction.	**M1** **M1** **A1** **A1** **A1** **A1**	Award this A1 for a correct concluding statement and correct presentation of proof byinduction

	METHOD 2 Let the result be true for $n = k$, ie $\mathbf{A}^k = k\mathbf{A} - (k-1)\mathbf{I}$ $= \begin{bmatrix} 1 & 0 \\ 2k & 1 \end{bmatrix}$ Consider, for $n = k + 1$, $\mathbf{A}^{k+1} = \begin{bmatrix} 1 & 0 \\ 2k & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ 2(k+1) & 1 \end{bmatrix}$ Hence true for $n = k \Rightarrow$ true for $n = k + 1$ and since trivially true for $n = 1$ ($\mathbf{A} = \mathbf{A}$), the result is proved by induction.	**M1** **A1** **M1** **A1** **A1** **A1**	 Award this A1 for a correct concluding statement and correct presentation of proof byinduction
	METHOD 3 Let the result be true for $n = k$, ie $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^k = k\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - (k-1)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Consider, for $n = k + 1$, $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^{k+1} = \left\{k\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - (k-1)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ 2k & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2(k+1) & 1 \end{bmatrix}$ But the assumed result for $n = k$ can be written as $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^k = \begin{bmatrix} 1 & 0 \\ 2k & 1 \end{bmatrix}$ Hence true for $n = k \Rightarrow$ true for $n = k + 1$ and since trivially true for $n = 1$ ($\mathbf{A} = \mathbf{A}$), the result is proved by induction.	 **M1** **M1** **A1** **A1** **A1** **A1**	 Award this A1 for a correct concluding statement and correct presentation of proof byinduction
9(a) **(b)**	Taking logs correctly, $\ln f(x) = x\ln 2 + \ln\sin x$ Differentiating, $\frac{f'(x)}{f(x)} = \ln 2 + \cot x$ $f'(x) = 2^x \sin x(\ln 2 + \cot x)$ Stationary value where $f'(x) = 0$ $x = \cot^{-1}(-\ln 2)$ cao $= 2.18$	 **M1** **A1A1** **A1** **M1** **A1** **A2**	 Condone ignoring sin$x = 0$ Award A1 for– 0.96

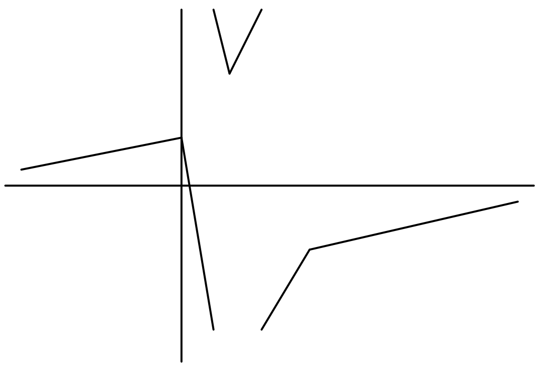
10(a)(i)	$\lvert z + 3\rvert = k\lvert z - \mathrm{i}\rvert$ Putting $z = x + \mathrm{i}y$, $(x+3)^2 + y^2 = k^2x^2 + k^2(y-1)^2$ $x^2 + 6x + 9 + y^2 = k^2x^2 + k^2y^2 - 2k^2y + k^2$ $(k^2-1)x^2 + (k^2-1)y^2 - 6x - 2k^2y + k^2 - 9 = 0$ (which is the equation of the circle.)	**M1** **A1** **A1**	
(ii)	Rewriting the equation in he form $x^2 + y^2 - \frac{6}{(k^2-1)}x - \frac{2k^2}{(k^2-1)}y = \frac{9-k^2}{(k^2-1)}$ Completing the square, $\left(x - \frac{3}{k^2-1}\right)^2 + \left(y - \frac{k^2}{k^2-1}\right)^2 = \text{terms involving } k$ Centre = $\left(\frac{3}{k^2-1}, \frac{k^2}{k^2-1}\right)$	**M1** **m1** **A1** **A1**	Award full credit for the use of the standard result for the coordinates of the centre
(b)(i)	$6x + 2y + 8 = 0$	**B1**	
(ii)	It is the perpendicular bisector of the line joining the points (– 3,0) and (0,1)	**B1**	

FP2

Ques	Solution	Mark	Notes
1(a)	Let		
	$\frac{5}{(x^2+1)(2-x)} = \frac{Ax+B}{x^2+1} + \frac{C}{2-x}$	**M1**	
	$= \frac{(Ax+B)(2-x)+C(x^2+1)}{(x^2+1)(2-x)}$		
	$A=1; B=2; C=1$	**A1A1A1**	
	$\left(\frac{5}{(x^2+1)(2-x)} = \frac{x+2}{(x^2+1)} + \frac{1}{2-x}\right)$		
(b)			
	$u = \tan x \Rightarrow \mathrm{d}u = \sec^2 x\mathrm{d}x$	**B1**	
	$[0,\pi/4] \to [0, 1]$	**B1**	
	$I = \int_0^1 \frac{5}{(2-u)} \times \frac{\mathrm{d}u}{(1+u^2)}$	**M1A1**	
	$= \int_0^1 (\frac{u+2}{u^2+1} + \frac{1}{2-u})\mathrm{d}u$	**A1**	
	$= \left[\frac{1}{2}\ln(u^2+1) + 2\tan^{-1}u - \ln(2-u)\right]_0^1$	**B1B1B1**	
	= 2.61 cao	**A1**	Award M0 if no working
2(a)	Denoting the two functional expressions by f_1, f_2		
	$f_1(-1) = 4,\ f_2(-1) = -a-b$	**M1**	
	Therefore $a + b = -4$	**A1**	
	$f_1'(x) = 2x-1,\ f_2'(x) = 3ax^2+b$	**M1**	
	$f_1'(-1) = -3,\ f_2'(1) = 3a+b$		
	Therefore $3a + b = -3$	**A1**	
	Solving, $a = \frac{1}{2}, b = -\frac{9}{2}$	**A1A1**	FT one slip in equations
(b)			FT if possible
	Solving $\frac{1}{2}x^3 - \frac{9}{2}x = 0; x = -3$	**M1A1**	Award M1 for attempting to solve this equation
3(a)	Modulus of cube roots = $\sqrt[3]{2}$	**B1**	
	$\mathrm{R1} = \sqrt[3]{2}(\cos\pi/4 + \mathrm{i}\sin\pi/4)$	**M1**	Use of de Moivre's Theorem
	= 0.891 + 0.891i	**A1**	FT their modulus
	$\mathrm{R2} = \sqrt[3]{2}(\cos 11\pi/12 + \mathrm{i}\sin 11\pi/12)$	**M1**	Addition of 2π/3 to argument
	$= -1.217 + 0.326\mathrm{i}$	**A1**	
	$\mathrm{R3} = \sqrt[3]{2}(\cos 19\pi/12 + \mathrm{i}\sin 19\pi/12)$		Penalise accuracy only once
	$= 0.326 - 1.217\mathrm{i}$	**A1**	

(b)(i) **(ii)**	z^n is real when n = 4 and imaginary when n = 2.	**B2** **B1**	Award B1 for n = 8
4	METHOD 1 Combining the first and third terms, $2\cos\left(2\theta+\frac{\pi}{6}\right)\cos\theta+\cos\left(2\theta+\frac{\pi}{6}\right)=0$ $\cos\left(2\theta+\frac{\pi}{6}\right)(2\cos\theta+1)=0$ Either $\cos\theta = -\frac{1}{2}$, $\theta=2n\pi\pm\frac{2\pi}{3}$ or $(2n+1)\pi\pm\frac{\pi}{3}$ Or $\cos\left(2\theta+\frac{\pi}{6}\right)=0$ $2\theta+\frac{\pi}{6}=2n\pi\pm\frac{\pi}{2}$ or $\left(n+\frac{1}{2}\right)\pi$ $\theta=n\pi\pm\frac{\pi}{4}-\frac{\pi}{12}$ or $\frac{n\pi}{2}+\frac{\pi}{6}$	 **M1A1** **A1** **M1** **A1** **M1** **A1** **A1**	 M1 for combining two terms Accept equivalent answers Accept equivalent answers
	METHOD 2 $\cos\theta\cos\frac{\pi}{6}-\sin\theta\sin\frac{\pi}{6}+\cos2\theta\cos\frac{\pi}{6}$ $-\sin2\theta\sin\frac{\pi}{6}+\cos3\theta\cos\frac{\pi}{6}-\sin3\theta\sin\frac{\pi}{6}=0$ Combining appropriate terms, $\cos\frac{\pi}{6}(2\cos\theta\cos2\theta+\cos2\theta)$ $=\sin\frac{\pi}{6}[2\sin2\theta\cos\theta+\sin2\theta]$ $\frac{\sqrt{3}}{2}\cos2\theta(2\cos\theta+1)=\frac{1}{2}\sin2\theta(2\cos\theta+1)$ Either $\cos\theta = -\frac{1}{2}$, $\theta=2n\pi\pm\frac{2\pi}{3}$ or $(2n+1)\pi\pm\frac{\pi}{3}$ Or $\tan2\theta=\sqrt{3}$ $2\theta=n\pi+\frac{\pi}{3}$ $\theta=\frac{n\pi}{2}+\frac{\pi}{6}$	 **M1** **A1** **A1** **M1** **A1** **M1** **A1** **A1**	 Accept equivalent answers Accept equivalent answers

5(a)	$\frac{d}{dx}\left(\int_0^x e^{\sqrt{u}}du\right) = e^{\sqrt{x}}$	**B1**	Do not accept integration followed by differentiation
(b)	Put $y = x^2; \frac{dy}{dx} = 2x$	**M1**	
	$\frac{d}{dx}\left(\int_0^{x^2} e^{\sqrt{u}}du\right) = \frac{d}{dy}\left(\int_0^{y} e^{\sqrt{u}}du\right) \times \frac{dy}{dx}$	**A1**	Do not accept integration followed by differentiation
	$= 2xe^x$	**A1**	
(c)	$\int_x^{x^2} e^{\sqrt{u}}du = \int_0^{x^2} e^{\sqrt{u}}du - \int_0^{x} e^{\sqrt{u}}du$	**M1**	Award this M1 for the difference of integrals
	$\frac{d}{dx}\left(\int_x^{x^2} e^{\sqrt{u}}du\right) = 2xe^x - e^{\sqrt{x}}$ cao	**A1**	
6(a)	We are given that		
	$x^2 + (y-3)^2 = (y+3)^2$	**M1**	Do not accept solutions which assume the equation given the focus and directrix
	$x^2 + y^2 - 6y + 9 = y^2 + 6y + 9$	**A1**	
	$x^2 = 12y$		
(b)(i)	$x^2 = 36t^2; 12y = 36t^2$	**B1**	
	showing that the point $(6t, 3t^2)$ lies on C.		
(ii)	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	
	$= \frac{6t}{6} = t$	**A1**	
	The equation of the tangent is		
	$y - 3t^2 = t(x - 6t)$	**M1A1**	
	$y = tx - 3t^2$		
(iii)	Substituting (0,– 12) into the equation,		
	$-12 = -3t^2$	**M1**	
	$t = \pm 2$	**A1**	
(iv)	Since the positive gradient of the tangent is equal to 2, the angle between the tangent and the y-axis is equal to $\tan^{-1}(1/2)$.	**M1**	Award M1 for any valid method
	The angle between the tangents is therefore equal to $2\tan^{-1}(1/2) = 53.1°$ or 0.927 rad	**A1**	Accept 126.9° or 2.21 rad

7(a)	$x = 1, x = 2$	**B1**	
(b)	$f(0) = 1$ giving the point (0,1) $f(x) = 0 \Rightarrow x = 2/3$ giving the point (2/3,0)	**B1** **M1A1**	
(c)	$f'(x) = -\frac{1}{(x-1)^2} + \frac{4}{(x-2)^2}$	**B1**	
	At a stationary point, $\frac{1}{(x-1)^2} = \frac{4}{(x-2)^2}$	**M1**	
	$\frac{1}{(x-1)} = \pm\frac{2}{(x-2)}$	**A1**	
	giving (0,1) and (4/3,9)	**A1A1**	Award A1A0 if only x values given
	$f''(x) = \frac{2}{(x-1)^3} - \frac{8}{(x-2)^3}$	**M1**	Accept any valid method including looking at appropriate values of $f(x)$ or $f'(x)$
	$f''(0) < 0$ so that (0,1) is a maximum $f''(4/3) > 0$ so that (4/3,9) is a minimum	**A1** **A1**	
(d)		**G1** **G1**	Award G1 for 2 correct branches
(e)(i)	$f(-1) = 5/6$; $f(0) = 1$ $f(S) = [5/6,1]$	**M1** **A1**	
(ii)	Solve $\frac{1}{x-1} - \frac{4}{x-2} = -1$	**M1**	
	$x^2 - 6x + 4 = 0$ $x = 3 \pm \sqrt{5}$ $f^{=1}(S) = [2/3, 3-\sqrt{5}] \cup [3+\sqrt{5}, \infty)$	**A1** **A1** **A1**	

Ques	Solution	Mark	Notes
1(a)	Expanding the right hand side, $5\cosh\theta + 3\sinh\theta = r\cosh\theta\cosh\alpha + r\sinh\theta\sinh\alpha$ Therefore $r\cosh\alpha = 5$ and $r\sinh\alpha = 3$ Squaring and subtracting, $r^2(\cosh^2\alpha - \sinh^2\alpha) = 5^2 - 3^2$ so that $r = 4$ Dividing, $\frac{\sinh\alpha}{\cosh\alpha} = \tanh\alpha = \frac{3}{5}$ $\alpha = \tanh^{-1}\left(\frac{3}{5}\right) = 0.693$	**M1** **A1** **A1** **A1**	
(b)	Substituting, $4\cosh(\theta + 0.693) = 10$ $(\theta + 0.693) = \pm\cosh^{-1}\left(\frac{10}{4}\right)$ $\theta = -0.693 \pm \cosh^{-1}\left(\frac{10}{4}\right)$ $= -2.26,\ 0.874$	**M1** **A1** **A1A1**	Condone the absence of ± here
2	EITHER $I = \int_0^{\pi/2} e^{2x} d(\sin x)$ $= \left[e^{2x}\sin x\right]_0^{\pi/2} - 2\int_0^{\pi/2} e^{2x}\sin x dx$ $= e^{\pi} - 2\int_0^{\pi/2} e^{2x} d(-\cos x)$ $= e^{\pi} + 2\left[e^{2x}\cos x\right]_0^{\pi/2} - 4I$ $= e^{\pi} - 2 - 4I$ $I = \frac{e^{\pi} - 2}{5}$	**M1** **A1** **A1A1** **A1** **A1** **A1**	

	OR $I = \int_0^{\pi/2} \cos x \, \mathrm{d}\left(\frac{e^{2x}}{2}\right)$ $= \left[\frac{e^{2x}}{2}\cos x\right]_0^{\pi/2} + \frac{1}{2}\int_0^{\pi/2} e^{2x}\sin x \mathrm{d}x$ $= -\frac{1}{2} + \frac{1}{2}\int_0^{\pi/2}\sin x \mathrm{d}\left(\frac{e^{2x}}{2}\right)$ $= -\frac{1}{2} + \frac{1}{4}\left[e^{2x}\sin x\right]_0^{\pi/2} - \frac{1}{4}I$ $= -\frac{1}{2} + \frac{1}{4}e^{\pi} - \frac{1}{4}I$ $I = \frac{e^{\pi}/4 - 1/2}{5/4} = \frac{e^{\pi} - 2}{5}$	**M1** **A1** **A1A1** **A1** **A1** **A1**	
3(a)(i) **(ii)** **(b)(i)** **(ii)**	$f'(x) = 12x^3 - 12x^2 - 6x - 6$ $f'(1.4) = -4.99...f'(1.6) = 2.83...$ The change in sign shows that α lies between 1.4 and 1.6. Since α satisfies $f'(\alpha) = 0$, it follows that $12\alpha^3 - 12\alpha^2 - 6\alpha - 6 = 0$ so that $2\alpha^3 = 2\alpha^2 + \alpha + 1$ $\alpha = \left(\frac{2\alpha^2 + \alpha + 1}{2}\right)^{\frac{1}{3}}$ Let $F(x) = \left(\frac{2x^2 + x + 1}{2}\right)^{\frac{1}{3}}$ $F'(x) = \frac{1}{3}\left(\frac{2x^2 + x + 1}{2}\right)^{-\frac{2}{3}} \times \left(\frac{4x + 1}{2}\right)$ $F'(1.5) = 0.506...$ The sequence converges because $\|F'(1.5)\| < 1$ Using the iterative formula, successive values are 1.5 1.518294486 1.527545210 etc α = 1.537 (to 3 dps)	**B1** **B1** **B1** **M1** **A1** **M1A1** **A1** **A1** **M1** **A1** **A1** **A1**	

4(a)	$f'(x) = \dfrac{\sinh x}{1+\cosh x}$	**B1**	
	$f''(x) = \dfrac{\cosh x(1+\cosh x) - \sinh^2 x}{(1+\cosh x)^2}$	**M1**	
	$= \dfrac{\cosh x + 1}{(1+\cosh x)^2}$	**A1**	
	$= \dfrac{1}{1+\cosh x}$		
(b)	$f'''(x) = -\dfrac{\sinh x}{(1+\cosh x)^2}$	**B1**	
	$f''''(x) = \dfrac{-\cosh x(1+\cosh x)^2 + \text{term inc sinh}x}{(1+\cosh x)^4}$	**M1A1**	
	$f(0) = \ln 2,\ f'(0) = 0,\ f''(0) = \dfrac{1}{2}$ $f'''(0) = 0,\ f''''(0) = -\dfrac{1}{4}$	**B1**	FT their derivatives
	The Maclaurin series for *f*(*x*) is $\ln 2 + \dfrac{x^2}{4} - \dfrac{x^4}{96} + \ldots$	**M1A1**	
5(a)	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = 1 + \cos t; \dfrac{\mathrm{d}y}{\mathrm{d}t} = \sin t$	**B1**	
	$\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = 1 + 2\cos t + \cos^2 t + \sin^2 t$	**M1**	
	$= 2(1+\cos t)$	**A1**	
	$= 4\cos^2 \dfrac{1}{2}t$		Convincing
(b)(i)	Arc length $= \displaystyle\int_0^{\pi} 2\cos \frac{1}{2}t\ \mathrm{d}t$	**B1**	
	$= \left[4\sin \dfrac{1}{2}t\right]_0^{\pi}$	**B1**	
	$= 4$	**B1**	

(ii)	CSA = $2\pi\int y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$	**M1**	
	$= 2\pi\int_0^{\pi}(1-\cos t)\times 2\cos\frac{1}{2}t\,dt$	**A1**	
	$= 4\pi\int_0^{\pi}(\cos\frac{1}{2}t\,dt-\frac{1}{2}\left(\cos\frac{3}{2}t+\cos\frac{1}{2}t\right))\,dt$	**A1**	Or $8\pi\int_0^{\pi}\sin^2\frac{1}{2}t\cos\frac{1}{2}t\,dt$
	$= 4\pi\left[\sin\frac{1}{2}t-\frac{1}{3}\sin\frac{3}{2}t\right]_0^{\pi}$	**A1**	$=\frac{16\pi}{3}\left[\sin^3\frac{1}{2}t\right]_0^{\pi}$
	$=\frac{16\pi}{3}$	**A1**	
6(a)	$\frac{d}{dx}\left((4-x^2)^{\frac{3}{2}}\right)=\frac{3}{2}(4-x^2)^{\frac{1}{2}}\times(-2x)$		
	$=-3x(4-x^2)^{\frac{1}{2}}$	**B1**	Convincing
(b)	$I_n=-\frac{1}{3}\int_0^2 x^{n-1}\frac{d}{dx}((4-x^2)^{3/2}\,dx$	**M1**	
	$=-\frac{1}{3}\left[x^{n-1}(4-x^2)^{3/2}\right]_0^2+\frac{n-1}{3}\int_0^2 x^{n-2}(4-x^2)^{3/2}dx$	**A1A1**	
	$=\left(\frac{n-1}{3}\right)\int_0^2 x^{n-2}(4-x^2)\sqrt{4-x^2}\,dx$	**A1**	
	$=\frac{n-1}{3}\left(4I_{n-2}-I_n\right)$	**A1**	
	$I_n=\left(\frac{4(n-1)}{n+2}\right)I_{n-2}$		
(c)(i)	Evaluate $I_0=\int_0^2\sqrt{4-x^2}\,dx$		
	Put $x=2\sin\theta, dx=2\cos\theta\,d\theta,[0,2]\to[0,\pi/2]$	**M1**	
	$I_0=4\int_0^{\pi/2}\cos^2\theta\,d\theta$	**M1A1**	
	$=2\int_0^{\pi/2}\left(1+\cos 2\theta\right)d\theta$	**A1**	
	$= 2\left[\theta+\frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$	**A1**	
	$=\pi$		
(ii)	$I_4=2I_2$	**M1**	
	$= 2\times1\times I_0$	**A1**	
	$= 2\pi$	**A1**	

7(a)	Consider $x = r\cos\theta$ $= \tan\left(\frac{\theta}{2}\right)\cos\theta$	**B1**	
	$\frac{dx}{d\theta} = \frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\cos\theta - \tan\left(\frac{\theta}{2}\right)\sin\theta$	**M1**	
	The tangent is perpendicular to the initial line where $\frac{1}{2}\sec^2\left(\frac{\theta}{2}\right)\cos\theta = \tan\left(\frac{\theta}{2}\right)\sin\theta$	**A1**	
	$\frac{1}{2}\left(1+\tan^2\left(\frac{\theta}{2}\right)\right) = \tan\left(\frac{\theta}{2}\right)\frac{\sin\theta}{\cos\theta}$	**A1**	
	$2\tan\theta\tan\left(\frac{\theta}{2}\right) = 1+\tan^2\left(\frac{\theta}{2}\right)$ Putting $t = \tan\left(\frac{\theta}{2}\right)$,		
	$2t \times \frac{2t}{1-t^2} = 1+t^2$	**M1**	
	$t^4 + 4t^2 - 1 = 0$	**A1**	
	$t^2 = -2+\sqrt{5}$ $\left(t = \sqrt{-2+\sqrt{5}}\right)$	**A1**	
	$\theta = 0.905$ (51.8°)	**A1**	
	$r = t = 0.486$	**A1**	
(b)	Area = $\frac{1}{2}\int r^2 d\theta$		
	$= \frac{1}{2}\int_0^{\pi/2} \tan^2\frac{\theta}{2}\, d\theta$	**M1**	
	$= \frac{1}{2}\int_0^{\pi/2} (\sec^2\frac{\theta}{2} - 1)\, d\theta$	**A1**	
	$= \frac{1}{2}\left[2\tan\frac{\theta}{2} - \theta\right]_0^{\pi/2}$	**A1**	
	$= 1 - \frac{\pi}{4}$ (0.215)	**A1**	

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