Rewarding Learning

ADVANCED
General Certificate of Education
2022 Reserve Series

Centre Number

Candidate Number

Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

AMT11

[AMT11]
MONDAY 27 JUNE, MORNING

TIME

2 hours 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all ten** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
Show clearly the full development of your answers. **Answers without working may not gain full credit.**
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

1 **(a)** State whether the following sequences converge, diverge or oscillate.

(i) $\dfrac{2n^2}{n+3}$ [1]

(ii) $\cos\left(\dfrac{n\pi}{3}\right)$ [1]

(iii) $\dfrac{5n}{3n-1}$ [1]

(b) Find $\sum_{r=1}^{\infty} 3\left(\frac{2}{5}\right)^r$ [5]

(c) The first three terms of an arithmetic progression are

$$x, \; 3x + 1, \; 3x^2 \qquad \text{where } x > 0$$

(i) Find the value of x. [4]

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(ii) Find the sum of the first 10 terms of this arithmetic progression. [4]

2 The shaded region R, as shown in **Fig. 1** below, is the major segment of a circle of radius r cm with angle AOB = $\frac{\pi}{3}$ radians.

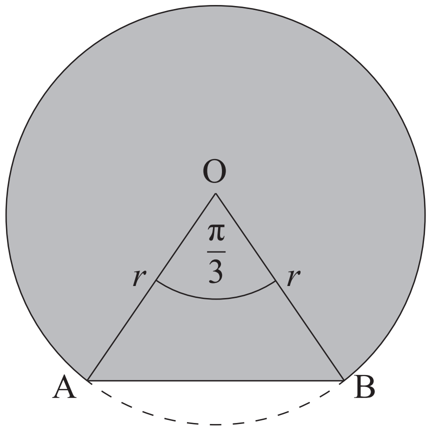

Fig. 1

(i) Find the perimeter of R in terms of r. [3]

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The perimeter of R is $(10\pi + 6)$ cm.

(ii) Find the value of r. [2]

(iii) Find the area of R. [5]

3 **(a)** Simplify as far as possible

$$\frac{4x^2 - 25}{3x^2 + 14x + 8} \div \frac{6x - 15}{x + 4}$$ [5]

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(b) Find, in ascending powers of x, the expansion of

$$\frac{x + 3}{\sqrt{x + 2}}$$

up to and including the term in x^2 [6]

[Turn over

4 **(a)** The graph of the function $y = f(x)$ is shown in **Fig. 2** below.

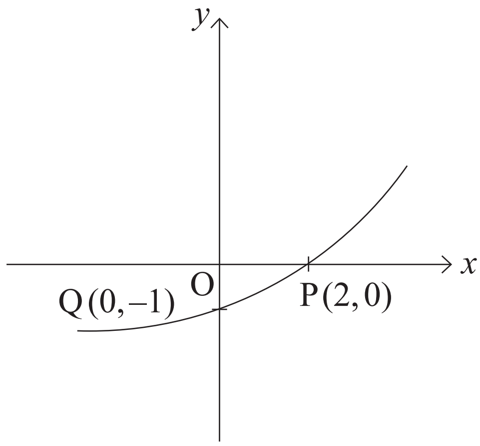
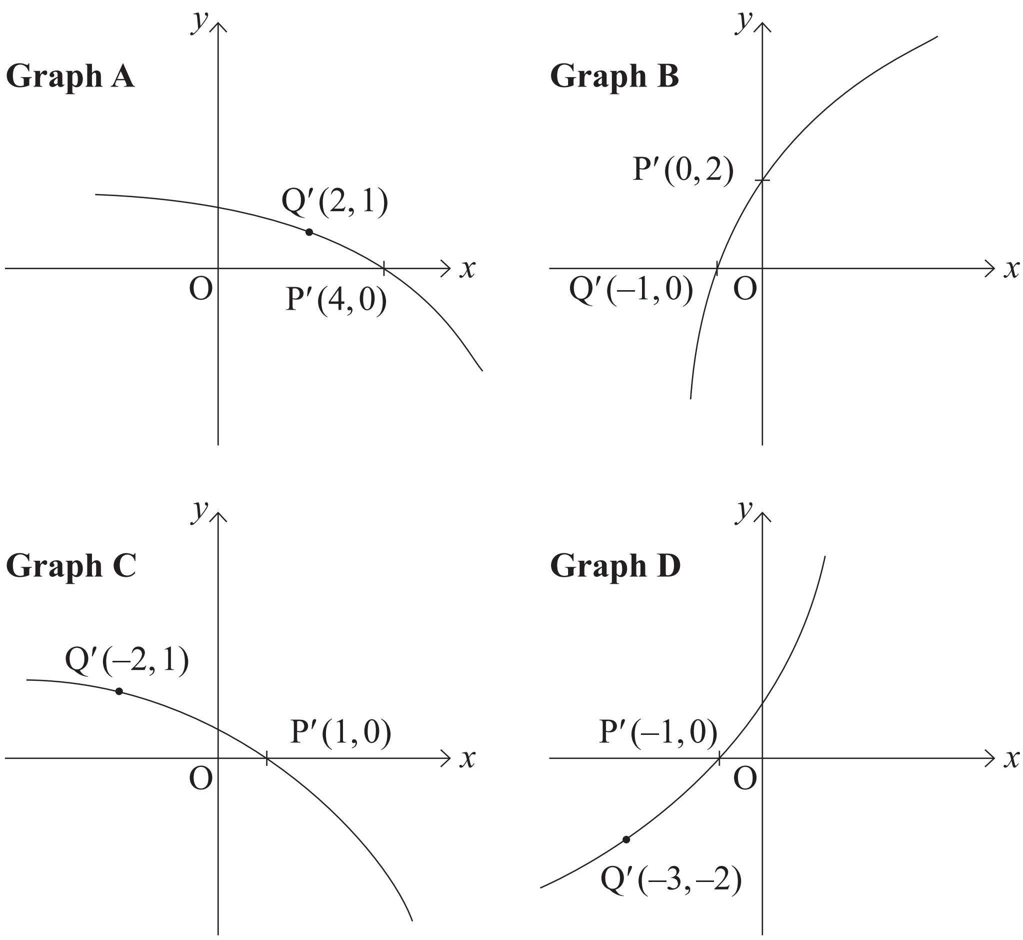


Fig. 2

The curve cuts the axes at P(2, 0) and Q(0, –1).

Fig. 3 below shows five different transformations of $y = f(x)$.



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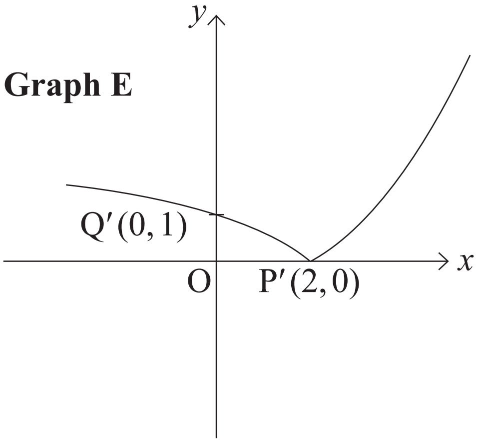

Fig. 3

Complete the following statements:

(i) $y = 2\mathrm{f}(x + 3)$ is represented by Graph ………………… [1]

(ii) $y = -\mathrm{f}(x - 2)$ is represented by Graph ………………… [1]

(iii) $y = |\mathrm{f}(x)|$ is represented by Graph ………………… [1]

(iv) $y = \mathrm{f}^{-1}(x)$ is represented by Graph ………………… [1]

(b) The functions g and h are defined by:

$$g(x) = 2 + \cos x \qquad x \in \mathbb{R} \quad 0 \leqslant x \leqslant \pi$$

$$h(x) = \frac{1}{1 + x} \qquad x \in \mathbb{R} \quad x \neq -1$$

(i) State the range of $g(x)$. [2]

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(ii) Find the inverse function $h^{-1}(x)$, stating its domain. [5]

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(iii) Find the composite function hg(x), stating its domain. [4]

[Turn over

5 The points of intersection of the curves

$$y = \operatorname{cosec}^2 3x$$

and

$$y = x^2 + 1$$

can be found by solving the equation

$$\operatorname{cosec}^2 3x - x^2 - 1 = 0$$

(i) Show that this equation has a root between $x = 0.3$ and $x = 0.5$ [3]

It is known that there is only one root α in the interval [0.3, 0.5].

(ii) By taking $x = 0.3$ as a first approximation and using the Newton-Raphson method once, find a better approximation to α. [6]

6 **(a)** Prove that

$$\frac{\cos 2\theta - \cos \theta + 1}{\sin 2\theta - \sin \theta} \equiv \cot \theta$$ [6]

(b) Solve the equation

$$\tan(\theta - 45°) = 6 \tan \theta \qquad \text{where } 0° \leqslant \theta \leqslant 360°$$ [7]

[Turn over

7 **(a)** A curve is defined by the parametric equations

$$x = \sin t + \cos t$$
$$y = 4 - 3 \sin 2t$$

Find the Cartesian equation of this curve. [7]

(b) A curve is given by the equation

$$3x + 5xy^2 - 16 = 2(x + y)^2$$

Find the equation of the tangent to the curve at the point $(1, 3)$.
Leave your answer in the form $ax + by + c = 0$, where a, b and c are integers. [9]

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8 **(a)** The curved surface of a glass bowl can be modelled by rotating the curve

$$y = 3 \tan 4x$$

between $x = 0$ and $x = \frac{\pi}{16}$ through 2π radians about the x-axis.

Find, in terms of π, the maximum volume of liquid that the bowl can contain. [9]

(b) Find $\int x\mathrm{e}^{2x}\,\mathrm{d}x$ [7]

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(c) Using the substitution

$$u = x - 3$$

find the exact value of

$$\int_4^6 \frac{x(x-5)}{(x-3)^2}\,dx$$ [9]

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9 **(i)** Show that $(x + 3)$ is a factor of

$$2x^3 + 11x^2 + 12x - 9$$ [3]

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(ii) Hence find

$$\int \frac{x^2 + 3x + 35}{2x^3 + 11x^2 + 12x - 9}\,dx$$ [15]

10 In a factory, a biological substance is placed in a large tank.

During the production process, the biological substance reproduces at a rate of $0.25A$ kg per hour, where A kg is the amount of substance present at time t hours.

At the same time, the biological substance is pumped out of the tank at a constant rate of 50 kg per hour.

(i) By setting up and solving a suitable differential equation, show that

$$A = 4ke^{0.25t} + 200$$ [7]

where k is a constant.

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At time $t = 0$, $A = 190$

(ii) Show that

$$A = 200 - 10e^{0.25t}$$ [2]

[Turn over

The tank must be completely empty by the end of each production shift.

(iii) Find the minimum length of time for each shift.
Note that a production shift always lasts for a whole number of hours. [3]

The factory management decides to change to 8-hour production shifts.

They plan to manage this by simply reducing the amount of biological substance in the tank at time $t = 0$

(iv) Find the maximum amount of biological substance at time $t = 0$ which makes this change feasible, giving your answer to 1 decimal place. [5]

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THIS IS THE END OF THE QUESTION PAPER

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For Examiner's use only	
Question Number	Marks
1	
2	
3	
4	
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6	
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8	
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10	
Total Marks	

Examiner Number