New Specification

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education

Mathematics

Assessment Unit AS 2

assessing

Applied Mathematics

[SMT21]

Assessment

MARK SCHEME

(Including Supplementary Mark Scheme to support Teachers)

General Marking Instructions

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

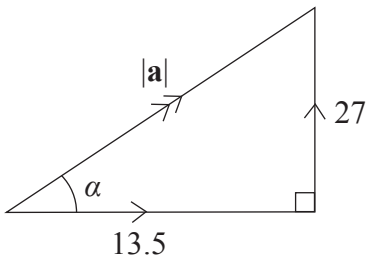
Some common examples of this occur in the following cases:

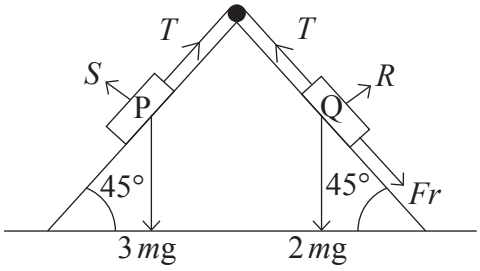
(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

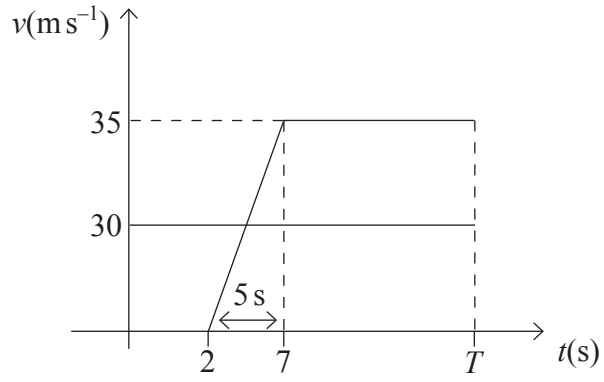
(b) readings taken from a candidate's inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A

				AVAILABLE MARKS
1	**(i)** $u = 30.4\,\text{m s}^{-1}$ $t = 2\,\text{s}$ $a = -\text{g}$ $v = ?$	$v = u + at$ $v = 30.4 + (-\text{g})(2)$ $v = 10.8\,\text{m s}^{-1}$	M1 W1	
	(ii) $u = 30.4\,\text{m s}^{-1}$ $a = -\text{g}$ $v = 0$ $s = ?$	$v^2 = u^2 + 2as$ $0^2 = 30.4^2 + 2(-\text{g})s$ $2\text{g}s = 924.16$ $s = 47.2\,\text{m}$	M1 M1 W1	
	(iii) Ball is treated as a particle No air resistance No other forces act on the ball Any other suitable assumption		MW1	6
2	**(i)** If in equilibrium Resultant force = 0			
	$\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = 0$ $3\mathbf{i} + 6\mathbf{j} + 2\mathbf{i} - 7\mathbf{j} + \mathbf{F_3} = 0$ $5\mathbf{i} - \mathbf{j} + \mathbf{F_3} = 0$ $\mathbf{F_3} = (-5\mathbf{i} + \mathbf{j})\text{N}$		M1 W1 MW1	
	(ii) $\mathbf{F_1} = 30\mathbf{i} + 60\mathbf{j}$ $\mathbf{F_2} = 2\mathbf{i} - 7\mathbf{j}$ $\mathbf{F_3} = -5\mathbf{i} + \mathbf{j}$		MW1	
	RF = ma $30\mathbf{i} + 60\mathbf{j} + 2\mathbf{i} - 7\mathbf{j} - 5\mathbf{i} + \mathbf{j} = 2\mathbf{a}$ $27\mathbf{i} + 54\mathbf{j} = 2\mathbf{a}$ $\mathbf{a} = (13.5\mathbf{i} + 27\mathbf{j})\,\text{m s}^{-2}$		M1W1 MW1	
				
	$\|\mathbf{a}\| = \sqrt{13.5^2 + 27^2}$ $= 30.2\,\text{m s}^{-2}$		M1W1	
	$\tan\alpha = \frac{27}{13.5}$ $\alpha = 63.4°$		MW1	10

			AVAILABLE MARKS
3	**(i)** 	MW2	
	(ii) P	M1	
	R (parallel to the plane)		
	$3mg \sin 45° - T = 3ma$	W1	
	Q		
	R (parallel to the plane)		
	$T - Fr - 2mg \sin 45° = 2ma$	W1	
	R (perpendicular to plane)		
	$R = 2mg \cos 45°$	W1	
	$Fr = 2\mu mg \cos 45°$	MW1	
	$3mg\left(\frac{1}{\sqrt{2}}\right) - 3ma - \mu\left(2mg\left(\frac{1}{\sqrt{2}}\right)\right) - 2mg\left(\frac{1}{\sqrt{2}}\right) = 2ma$	M1	
	$\frac{3g}{\sqrt{2}} - 3a - \frac{2\mu g}{\sqrt{2}} - \frac{2g}{\sqrt{2}} = 2a$		
	$\frac{g}{\sqrt{2}} - \frac{2\mu g}{\sqrt{2}} = 5a$		
	$a = \frac{g - 2\mu g}{5\sqrt{2}}$	W1	
	(iii) $1.3 = \frac{g - 2\mu g}{5\sqrt{2}}$	M1	
	$1.3\left(5\sqrt{2}\right) = g(1 - 2\mu)$		
	$\mu = 0.0310$	W1	11

			AVAILABLE MARKS
4	$v = u + at$ $35 = 0 + 7t$ $t = 5\,\text{s}$	MW1	
		MW2	
	Area under curve = distance	M1	
	Car $30T = d$	MW1	
	Police $d = \frac{1}{2}(T - 2 + T - 7)(35)$ $d = 35T - 157.5$	MW1	
	$30T = 35T - 157.5$ $-5T = -157.5$ $T = 31.5\,\text{s}$	M1 W1	8
		Section A	**35**

Section B: Statistics

AVAILABLE MARKS

5 **(i)** $S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 181397.65 - \frac{2512.8 \times 551.8}{8} = 8077.27$ MW1

$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 861132.58 - \frac{2512.8^2}{8} = 71862.1$ MW1

$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n} = 39042.42 - \frac{551.8^2}{8} = 982.015$ MW1

$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

$= \frac{8077.27}{\sqrt{71862.1 \times 982.015}}$ M1

$= 0.962$ W1

(ii) The greater the annual rainfall, the greater the yield of wheat. MW1

(iii) For every additional 1mm of rain, the yield of wheat increases by 0.112 bushels per hectare. MW2 8

6 **(a)** **(i)** $4c = 1 - (0.2 + 0.47 + 0.09)$ M1

$c = 0.06$ W1

(ii) $P(X < 5) = P(X = 3) + P(X = 4)$ M1

$= 0.2 + 0.06$

$= 0.26$ W1

(b) **(i)** $P(C) = \frac{1 - 0.64}{2} = 0.18$ MW1

(ii) $P(\overline{\text{Toast}} \cap \overline{\text{Tea}}) = 0.6 \times 0.82$ M1

$= 0.492$ W1 7

			AVAILABLE MARKS
7	**(i)** $\bar{t} = \frac{13\,600}{800}$	MW1	
	$= 17$ seconds	W1	
	(ii) The manager's concern is justified since the mean time is more than 15 seconds.	MW1	
	(iii) $s_t^2 = \frac{1}{799}\left[366\,600 - \frac{13\,600^2}{800}\right]$	MW1	
	$= 169.4618 \ldots$		
	$= 169$ seconds^2	W1	
	(iv) Three improvements, e.g.		
	Include times from a range of days in the sample (not just Tuesday)	MW1	
	Include times throughout the day.	MW1	
	Include times from days over a number of weeks.	MW1	8
8	**(i)** X = number of faulty components in the batch, so $X \sim \text{Bin}(20, 0.03)$	M1	
	$\text{P}(X = 2) = \binom{20}{2}(0.03)^2\,(0.97)^{18}$	M1	
	$= 0.0988$	W1	
	(ii) $X \sim \text{Bin}(20, 0.03)$		
	$\text{P}(X < 2) = \text{P}(X = 0) + \text{P}(X = 1)$		
	$= \binom{20}{0}(0.97)^{20} + \binom{20}{1}(0.03)^1\,(0.97)^{19}$	M1 W1	
	$= 0.880$	W1	
	(iii) Y = number of faulty components in the batch, so $Y \sim \text{Bin}(n, 0.03)$	M1	
	$\text{P}(Y = 0) = \binom{n}{0}(0.97)^n$		
	$= 0.97^n$	MW1	
	$0.97^n = 0.2957$	M1	
	$\log 0.97^n = \log 0.2957$	M1	
	$n \log 0.97 = \log 0.2957$		
	$n = \frac{\log 0.2957}{\log 0.97}$	MW1	
	$= 40.001 \ldots$		
	$= 40$	W1	12
		Section B	**35**
		Total	**70**

Breakdown of Marks

1 **(i)** M1 Use of a suitable equation of motion
W1 Correct answer

(ii) M1 Use of a suitable equation of motion
M1 Correctly placing final velocity = 0
W1 Correct answer

(iii) MW1 Any suitable assumption

2 **(i)** M1 Trying to use Sum of Forces = 0 (must be in vector form)
W1 Correctly adding all forces to sum = 0
MW1 Correct vector value for $\mathbf{F}_3$

(ii) MW1 Correctly increasing $\mathbf{F}_1$ by factor of 10
M1 Trying to use $\mathbf{F} = m\mathbf{a}$
W1 Correct values in $\mathbf{F} = m\mathbf{a}$
MW1 Correct vector solution for acceleration
M1 Attempt to use Pythagoras or Trig
W1 Correct magnitude
MW1 Correct value of angle (and direction)

Notes
[A] If substitute incorrect value of $\mathbf{F}_3$ from (i), can award
MW1 M1 W1(ft) MW1(ft) M1 W0 W0 i.e. max 5/7

3 **(i)** MW1 Correct forces on P
MW1 Correct forces on Q

Notes
[A] Tensions must be equal
[B] Weights must be shown as $3mg$ and $2mg$
[C] Arrows must be shown and in correct direction
[D] If T shown (as extra) on the pulley then just ignore
[E] Any other extra forces on either P or Q then the appropriate MW1 is not awarded

(ii) M1 Attempting to resolve in mutually perpendicular directions
W1 One resolving equation correct
W1 2nd resolving equation correct
W1 3rd resolving equation correct
MW1 Correct use of $F = \mu R$
M1 Trying to combine/solve equations
W1 Correct expression for μR (can be left in any form)

(iii) M1 Equate their expression for a to 1.3
W1 Correct value of μ given to 3sf

4 MW1 Correct time for acceleration of police car (may just be seen on graph)
MW1 Correct VT graph journey for car
MW1 Correct VT graph journey for police
M1 Use of area under curve = distance covered (even if any of earlier values incorrect)
MW1 Correct equation for car
MW1 Correct equation for police
M1 Equating their two expressions for distance
W1 Correct value for the time T

5 **(i)** MW1 $S_{xy} = 8077.27$
MW1 $S_{xx} = 71862.1$
MW1 $S_{yy} = 982.015$
M1 Trying their values of S_{xy}, S_{xx}, S_{yy} in $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$
W1 Correct answer

(ii) MW1 Correct statement about r (in context) – no ft from incorrect value in (i)
MW1 Correct statement about increase
MW1 Exact details given correctly

6 **(a)** **(i)** M1 Setting up correct equation in c
W1 Correct answer

(ii) M1 Trying to add P($X = 3$) and P($X = 4$)
W1 Correct answer

(b) **(i)** MW1 Correct answer

(ii) M1 Trying to multiply 2 probabilities
W1 Correct answer

7 **(i)** MW1 Dividing 13 600 by 800
W1 Correct answer

(ii) MW1 Correct statement

(iii) MW1 Trying correct expression for s_t^2
W1 Correct answer

Notes
[A] Use of $\dfrac{366\,000}{800} - 17^2$ is awarded 0/2

(iv) MW3 One mark for each of 3 correct and independent answers

8 **(i)** M1 Identifying binomial model
M1 Trying correct form of probability mass function to find P($X = 2$)
W1 Correct answer

Notes
[A] Use of calculator:
M1 – State X ~ Bin (20, 0.03)
M1 W1 – for correct answer

(ii) M1 Trying to set up an expression for P($X < 2$)
W1 Using correct values
W1 Correct answer

Notes
[A] Use of calculator:
M1 – State P($X < 2$)
W1 W1 – for correct answer

(iii) M1 Identifying Y ~ Bin (n, 0.03)
MW1 Correct expression for P ($X = 0$)
M1 Trying to set up the equation P ($X = 0$) = value given in question
M1 Knowing and trying to take logs on both sides
MW1 Correct expression for n (logs or unrounded decimal)
W1 Correct value of n

AVAILABLE MARKS