New Specification

Rewarding Learning

ADVANCED
General Certificate of Education

Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

[AMT11]

Assessment

MARK SCHEME

(Including Supplementary Mark Scheme to support Teachers)

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1 **(i)**	$300\pi = 4 \times (\frac{1}{2}(30)^2(\theta))$	M1 W1	
	$300\pi = 1800\theta$		
	$\theta = \dfrac{\pi}{6}$	W1	
(ii)	$P = 4\left(\dfrac{\pi}{6}\right)(30) + 8(30)$	M1 MW1	
	$P = (20\pi + 240)$ cm	W1	6
2 **(a)** **(i)**	converges	MW1	
(ii)	oscillates	MW1	
(b) **(i)**	$(1-3x)^{\frac{1}{2}}$	MW1	
	$= 1 + \left(\dfrac{1}{2}\right)(-3x) + \dfrac{(\frac{1}{2})(\frac{-1}{2})(-3x)^2}{2!} + \dfrac{(\frac{1}{2})(\frac{-1}{2})(\frac{-3}{2})(-3x)^3}{3!} + \ldots \ldots$	M1 W2	
	$= 1 - \dfrac{3}{2}x + \left(\dfrac{-1}{8}\right)(9x^2) + \left(\dfrac{1}{16}\right)(-27x^3) + \ldots \ldots$		
	$= 1 - \dfrac{3}{2}x - \dfrac{9}{8}x^2 - \dfrac{27}{16}x^3 + \ldots \ldots$	MW1	
(ii)	$\lvert 3x \rvert < 1$		
	$-\dfrac{1}{3} < x < \dfrac{1}{3}$	MW1	8

					AVAILABLE MARKS
3	**(a)**				

x 2 3 4 — MW1

y −0.8322... −2.9699... −2.6145... — MW1

$$\int_2^4 x\cos x\,dx \approx \frac{1}{2}(-0.8322\ldots + 2(-2.9699\ldots) + -2.6145\ldots)$$ M1 MW1

$$\approx \frac{1}{2}(-9.386823\ldots)$$

≈ -4.69 (3 sf) — W1

(b) $x^2 - 6 = 4x - x^2$ — M1

$2x^2 - 4x - 6 = 0$

$2(x - 3)(x + 1) = 0$

$x = 3, x = -1$ — W1

$$\int_{-1}^{3} (4x - x^2) - (x^2 - 6)\,dx$$ M1 W1

$$\int_{-1}^{3} (4x - 2x^2 + 6)\,dx$$

$$= \left[2x^2 - \frac{2x^3}{3} + 6x\right]_{-1}^{3}$$ W1

$$= (18 - 18 + 18) - \left(2 + \frac{2}{3} - 6\right)$$ M1

$$= \frac{64}{3}$$

$= 21.3$ (3 sf) — W1 — **12**

4 **(a)** **(i)** $5\sec^4(2x)(2)\sec(2x)\tan(2x)$ — M1 W2

$10\sec^5(2x)\tan(2x)$ — W1

(ii) $$\frac{e^{4x}(-\text{cosec}^2 x) - 4e^{4x}(\cot x)}{(e^{4x})^2}$$ M1 W2

$$\frac{-e^{4x}(\text{cosec}^2 x + 4\cot x)}{(e^{4x})^2}$$

$$\frac{-(\text{cosec}^2 x + 4\cot x)}{e^{4x}}$$ MW1

(b) $$V = \pi\int_0^4 (\sqrt{x} + 2)^2\,dx$$ M1

$$= \pi\int_0^4 (x + 4\sqrt{x} + 4)\,dx$$ W1

$$= \pi\int_0^4 (x + 4x^{\frac{1}{2}} + 4)\,dx$$

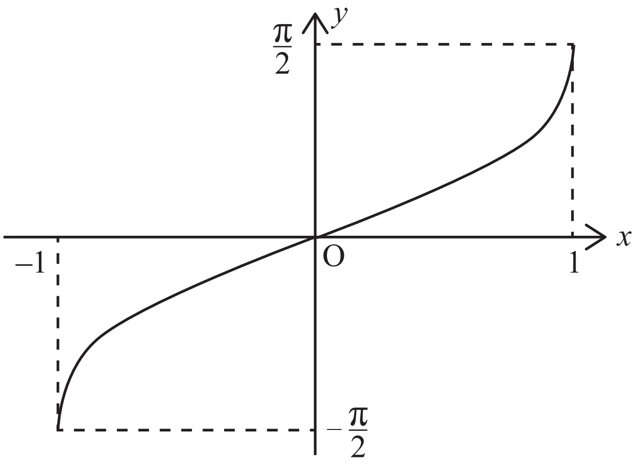
$$= \pi\left[\frac{x^2}{2} + \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + 4x\right]_0^4$$ M1 W2

$$= \pi\left[\left(\frac{16}{2} + \frac{8(8)}{3} + 16\right) - (0)\right]$$ M1

$$= \frac{136\pi}{3}$$ W1 — **15**

		AVAILABLE MARKS
5 **(i)** $\dfrac{3x^2-10x+5}{(x+1)(x-2)^2} \equiv \dfrac{A}{(x+1)} + \dfrac{B}{(x-2)} + \dfrac{C}{(x-2)^2}$	M1 W1	
$3x^2 - 10x + 5 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$	M1	
$x = 2$: $3C = 12 - 20 + 5$ $C = -1$	M1	
$x = -1$: $9A = 3 + 10 + 5$ $A = 2$		
$x = 0$: $4A - 2B + C = 5$ $B = 1$		
$\dfrac{2}{(x+1)} + \dfrac{1}{(x-2)} - \dfrac{1}{(x-2)^2}$	W2	
(ii) $\displaystyle\int_0^1 \frac{3x^2-10x+5}{(x+1)(x-2)^2}\,dx$		
$= \displaystyle\int_0^1 \frac{2}{(x+1)} + \frac{1}{(x-2)} - \frac{1}{(x-2)^2}\,dx$	M1	
$= \left[2\ln\lvert x+1\rvert + \ln\lvert x-2\rvert - \dfrac{(x-2)^{-1}}{-1}\right]_0^1$	M1 W2	
$= (2\ln 2 + \ln 1 - 1) - (2\ln 1 + \ln 2 - \tfrac{1}{2})$	M1	
$= \ln 2 - \dfrac{1}{2}$	MW1	12

6 **(a)** MW2



AVAILABLE MARKS

(b) $R\cos(x+\alpha) = R\cos x\cos\alpha - R\sin x\sin\alpha$ M1

$R\cos\alpha = 1 \qquad R\sin\alpha = \sqrt{3}$ M1

$\frac{R\sin\alpha}{R\cos\alpha} = \sqrt{3}$ M1

$\tan\alpha = \sqrt{3}$

$\alpha = 60°$ W1

$R^2(\sin^2\alpha + \cos^2\alpha) = 4$ M1

$R = 2$ W1

$\cos x - \sqrt{3}\sin x = 2\cos(x + 60°)$ MW1

(c) $\sin 3\theta = \sin(2\theta + \theta)$ M1

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$ M1

$= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$ M1

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$ W1

$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$ W1 14

Alternate solution

$\sin 3\theta = \sin(2\theta + \theta)$ M1

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$ M1

$= 2\sin\theta\cos\theta\cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$ M1

$= 2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$ W1

$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta(1 - \sin^2\theta) - \sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - \sin^3\theta - \sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$ W1

7 (a) $2x - 1 > 5$ $\quad$ $2x - 1 < -5$ M1

$x > 3$ or $x < -2$ W2 MW1

Alternate solution

$(2x - 1)^2 > 25$ M1

$4x^2 - 4x + 1 > 25$

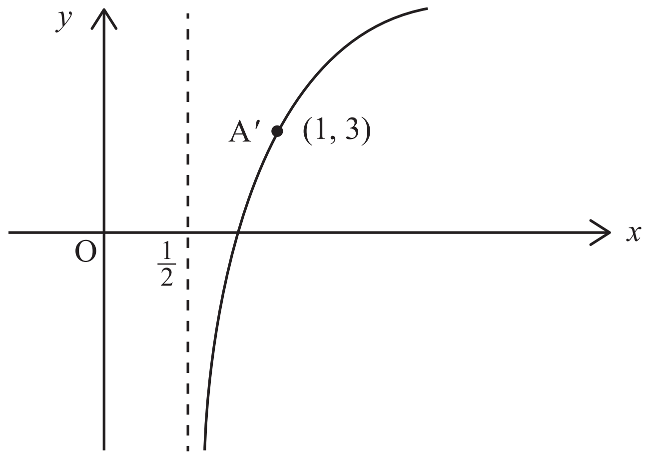
$4x^2 - 4x - 24 > 0$

$4(x^2 - x - 6) > 0$

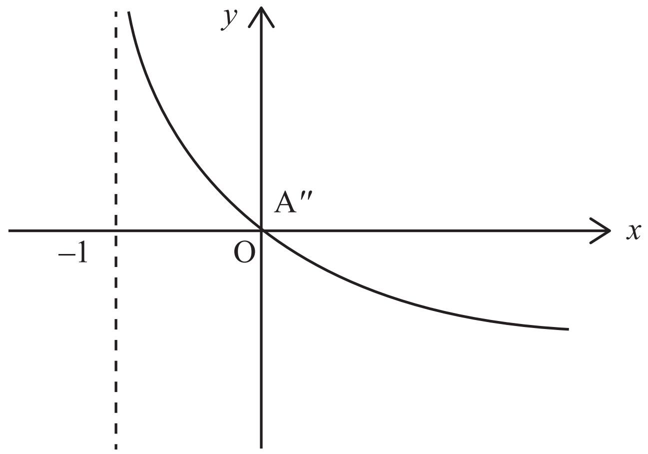
$4(x - 3)(x + 2) > 0$

$x > 3, x < -2$ W2 MW1

(b) (i) MW3



(ii) MW3



AVAILABLE MARKS

		AVAILABLE MARKS
(c) **(i)** $f(x) > 0$	MW1	
$g(x) > 1$	MW1	
(ii) $y = \frac{10}{x-1}$	M1	
$y(x-1) = 10$		
$yx - y = 10$		
$x = \frac{10 + y}{y}$	W1	
$f^{-1}(x) = \frac{10 + x}{x}$	MW1	
Domain $x \in \mathbb{R}, x > 0$	MW1	
(iii) $fg(x)$		
$f(e^{3x})$	M1	
$= \frac{10}{e^{3x} - 1}$	W1	
Domain $x \in \mathbb{R}, x > 0$	MW1	19
8 $2x + 5y\left(\frac{1}{x}\right) + 5\frac{dy}{dx}\ln x - (1)y^2 - x(2y)\frac{dy}{dx} = 0$	M2W5	
At (1, –1)		
$2 - 5 - 1 + 2\frac{dy}{dx} = 0$	M1	
$\frac{dy}{dx} = 2$	W1	
$y + 1 = 2(x - 1)$	M1	
$y = 2x - 3$	W1	11

			AVAILABLE MARKS
9 **(a)**	$\int 1 \times \ln x \, dx$	M1	
	$= x\ln x - \int x\left(\frac{1}{x}\right) dx$	M1 W2	
	$= x\ln x - x + c$	MW1	
(b)	$x = \frac{2}{3}\sin\theta$		
	$\frac{dx}{d\theta} = \frac{2}{3}\cos\theta$	M1 W1	
	$x = 0,\ \theta = 0$		
	$x = \frac{1}{3},\ \theta = \frac{\pi}{6}$	M1 W1	
	$\int_0^{\frac{\pi}{6}} \sqrt{4 - 9\left(\frac{2}{3}\sin\theta\right)^2}\left(\frac{2}{3}\cos\theta\right) d\theta$	M1 W2	
	$\int_0^{\frac{\pi}{6}} \sqrt{4 - 4\sin^2\theta}\left(\frac{2}{3}\cos\theta\right) d\theta$		
	$\int_0^{\frac{\pi}{6}} \sqrt{4\cos^2\theta}\left(\frac{2}{3}\cos\theta\right) d\theta$		
	$\int_0^{\frac{\pi}{6}} \frac{4}{3}\cos^2\theta \, d\theta$	W1	
	$\int_0^{\frac{\pi}{6}} \frac{4}{3}\left(\frac{1}{2}(\cos 2\theta + 1)\right) d\theta$	M1 W1	
	$\frac{2}{3}\int_0^{\frac{\pi}{6}} (\cos 2\theta + 1) d\theta$		
	$= \frac{2}{3}\left[\frac{\sin 2\theta}{2} + \theta\right]_0^{\frac{\pi}{6}}$	MW1	
	$= \frac{2}{3}\left(\left(\frac{\sqrt{3}}{4} + \frac{\pi}{6}\right) - (0 + 0)\right)$		
	$= \frac{\sqrt{3}}{6} + \frac{\pi}{9}$		
	$= 0.638$ (3 sf)	MW1	17

		AVAILABLE MARKS
10 **(i)** $\frac{dy}{dx} = -\sqrt{2}\,e^{-x}\sin x - \sqrt{2}\,e^{-x}\cos x$	M1 W2	
Stationary points		
$-\sqrt{2}\,e^{-x}\sin x - \sqrt{2}\,e^{-x}\cos x = 0$	M1	
$e^{-x} \neq 0$	MW1	
$\sin x + \cos x = 0$	M1	
$\tan x = -1$ (provided $\cos x \neq 0$)	MW1	
$x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$	W1	
(ii) $y = -e^{\frac{-3\pi}{4}},\ e^{\frac{-7\pi}{4}},\ -e^{\frac{-11\pi}{4}},\ e^{\frac{-15\pi}{4}}$	M1 W1	
$r = \frac{e^{\frac{-7\pi}{4}}}{-e^{\frac{-3\pi}{4}}} = -e^{-\pi}$		
$r = \frac{e^{-\frac{15\pi}{4}}}{-e^{-\frac{11\pi}{4}}} = -e^{-\pi}$	M1 W1	
(iii) $S_\infty = \frac{-e^{\frac{-3\pi}{4}}}{1 + e^{-\pi}}$	M1	
$= \frac{-e^{\frac{\pi}{4}}}{e^{\pi} + 1}$	W1	14

AVAILABLE MARKS

11 (i) $\frac{dx}{dt} = 4t + 4$

$\frac{dy}{dt} = 2\cos 2t$ MW2

$\frac{dy}{dx} = \frac{2\cos 2t}{4t + 4}$ M1

At stationary point:

$\frac{2\cos 2t}{4t + 4} = 0$ M1

Since $0 < t < \frac{\pi}{2} \Rightarrow t \neq -1$

$2\cos 2t = 0$

$t = \frac{\pi}{4}$ W1

$x = \frac{\pi^2}{8} + \pi, \; y = 1$

$x = 4.38, \; y = 1$ MW1

(ii) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{2\cos 2t}{4t + 4}\right)$ M1

$= \frac{d}{dt}\left(\frac{2\cos 2t}{4t + 4}\right) \times \frac{dt}{dx}$ M1

$= \left(\frac{(-4\sin 2t)(4t + 4) - 4(2\cos 2t)}{(4t + 4)^2}\right) \times \frac{1}{4t + 4}$ M1 W2

$t = \frac{\pi}{4}$:

$\frac{d^2y}{dx^2} = \left(\frac{-4(\pi + 4)}{(\pi + 4)^2}\right) \times \frac{1}{\pi + 4}$

$= \frac{-4}{(\pi + 4)^2} < 0$

$\left(\frac{\pi^2}{8} + \pi, 1\right)$ Maximum stationary point. MW1 12

		AVAILABLE MARKS
12 $\dfrac{dV}{dt} \propto -\sqrt{V}$		
$\dfrac{dV}{dt} = -k\sqrt{V}$	M1	
$\displaystyle\int \frac{1}{\sqrt{V}}\,dV = \int -k\,dt$	M1 W1	
$2V^{\frac{1}{2}} = -kt + c$	M1 W1	
$t = 0,\ V = V_0$: $\quad 2\sqrt{V_0} = c$	MW1	
$2\sqrt{V} = -kt + 2\sqrt{V_0}$		
$t = 1,\quad V = \frac{4}{9}V_0$:	M1	
$2\sqrt{\dfrac{4V_0}{9}} = -k + 2\sqrt{V_0}$		
$k = \dfrac{2\sqrt{V_0}}{3}$	W1	
$\sqrt{V} = \sqrt{V_0} - \dfrac{\sqrt{V_0}}{3}t$		
When empty, $V = 0$:	M1	
$\sqrt{V_0} - \dfrac{\sqrt{V_0}}{3}t = 0$		
$t = 3$ hours	W1	10
	Total	**150**

			AVAILABLE MARKS
1 **(i)**	Tries equation for area – must have evidence of using $\frac{1}{2}r^2\theta$ in equation	M1	
	Correct equation for θ	W1	
	Correct θ (must be in radians)	W1	
(ii)	Tries equation for arc length i.e. evidence of using $r\theta$ (allow ft for their θ in radians)	M1	
	Uses $8r$	MW1	
	Correct answer	W1	6
2 **(a)** **(i)**	Converges	MW1	
(ii)	Oscillates	MW1	
(b) **(i)**	Writes as $(1 - 3x)^{\frac{1}{2}}$	MW1	
	Correct structure of binomial	M1	
	Coefficients all correct – not necessary to be simplified at this stage	W1	
	Powers of $(-3x)$ all correct	W1	
	All terms correct	MW1	
(ii)	Correct range	MW1	8
3 **(a)**	Correct x values	MW1	
	Correct y values	MW1	
	Tries to use trapezium rule	M1	
	h correct	MW1	
	Correct answer	W1	
	Note [A] Incorrect value for h – can only award M1 provided the set-up of the trapezium rule is correct for their x, y, h values i.e. max 1/5		
(b)	Tries to find x values by equating $x^2 - 6$ and $4x - x^2$	M1	
	Correct x values	W1	
	Tries to integrate	M1	
	Subtracts equations	W1	
	Integration correct	W1	
	Tries to use limits	M1	
	Correct area	W1	12
	Notes [A] Limits substituted in opposite order and leaves answer as –21.3 Lose final W1 mark only i.e. max 6/7 [B] Subtracts equations in wrong order and leaves answer as –21.3 M1 W1 M1 W1 W1(ft) M1 W0 i.e. 6/7 max [C] Incorrect limits found – M1 W0 M1 W1 W1 M1 W0 i.e. 5/7 max		

					AVAILABLE MARKS
4	(a)	(i)	Tries to use function of a function	M1	
			$5\sec^4(2x)$	W1	
			$2\sec(2x)\tan(2x)$	W1	
			Correct answer	W1	
		(ii)	Tries to use quotient rule correctly	M1	
			$4e^{4x}$	W1	
			$-\mathrm{cosec}^2 x$	W1	
			Correct answer in simplified form	MW1	
	(b)		Tries to use volume formula	M1	
			Correct expansion	W1	
			Tries to integrate their expansion (conditional on 1st M1 being awarded)	M1	
			2 terms correct	W1	
			All terms correct	W1	
			Substitutes limits	M1	
			Correct answer in exact form	W1	15

Notes

[A] If use $V = \pi\int x^2\,dy$, can award M1 and nothing further

5	(i)	Tries to write as partial fractions	M1	
		Correct	W1	
		Equates numerators	M1	
		Substitutes x values	M1	
		2 values correct (cao)	W1	
		3rd value correct (cao)	W1	

Notes

[A] If omit $\frac{B}{x-2}$ can potentially award M1 W0 M1 M1 W0 W0 i.e. 3/6 max

	(ii)	Sets up integration as partial fractions	M1	
		Tries to integrate	M1	
		Both ln terms correct	W1	
		$(x-2)^{-1}$	W1	
		Tries to substitute limits	M1	
		Correct answer as shown	MW1	12

Notes

[A] Incorrect answer from (i)
Award M1 M1 W1(if there are two correctly integrated ln terms)
W1 (provided similar form of reciprocal term) M1 MW0 i.e. max of 5/6
[B] If correct answer just appears and no working, then assume calculator was used – can award M1 for trying to use calculator and MW1 for correct answer i.e. 2/6 max
[C] If incorrect answer just appears without working then 0/6

			AVAILABLE MARKS
6 **(a)**	Correct shape	MW1	
	$\pm \frac{\pi}{2}$ labels	MW1	
(b)	Tries to use $\cos(x + \alpha) \equiv \cos x \cos \alpha - \sin x \sin \alpha$	M1	
	Tries to equate coefficients	M1	
	Tries to find α	M1	
	Correct value of α	W1	
	Tries to find R	M1	
	Correct value of R	W1	
	Written in correct form	MW1	

Notes

[A] If 1st 2 lines not explicitly stated but remainder of working correct, can award all marks

[B] If 1st 2 lines not explicitly stated and tan α incorrect – M0 M0 M0 W0

Then award M1 W1 if R found correctly using Pythagoras
M0 W0 if R found incorrectly using incorrect trig
MW0 since final answer incorrect

i.e. max of 2/7

(c)	Writes as $(2\theta + \theta)$	M1	
	Tries to use sin (A + B)	M1	
	Uses double angle formulae for sine and cosine	M1	
	Changes all terms to powers of $\sin \theta$	W1	
	Correct final answer	W1	14

				AVAILABLE MARKS
7	**(a)**	Splits modulus into 2 inequalities **or** squares both sides	M1	
		[1] for each correct x value	W2	
		Correct inequalities (cao)	MW1	
	(b) (i)	Correct x value	MW1	
		Correct y value	MW1	
		Correct asymptote	MW1	
	(ii)	Crosses at origin	MW1	
		Correct asymptote	MW1	
		Correct shape i.e. evidence of reflection in x-axis	MW1	
	(c) (i)	Correct range for f	MW1	
		Correct range for g	MW1	
		Notes [A] Must be written as $f(x) > 0$ and $g(x) > 1$ i.e. $x > 0$ or $y > 0$ is awarded MW0		
	(ii)	Writes as $y = \ldots$ and then tries to change subject	M1	
		Correct $x =$	W1	
		Correct answer and notation	MW1	
		Correct domain	MW1	
		Notes [A] Accept either $f^{-1}(x) = \cdots$ or $f^{-1}: x \to \cdots$ formats [B] Award final MW1 even if $x \in \mathbb{R}$ is omitted in domain		
	(iii)	Tries to find composite	M1	
		Correct $fg(x)$	W1	
		Correct domain	MW1	19
		Notes [A] Accept either $fg(x) = \cdots$ or $fg: x \to \cdots$ formats [B] Award final MW1 even if $x \in \mathbb{R}$ is omitted in domain		

			AVAILABLE MARKS
8	Evidence of trying to use product rule	M1	
	Evidence of trying to use implicit differentiation	M1	
	One mark for each correct term	W5	
	Tries to substitute (1, –1) into their derivative	M1	
	Correct gradient	W1	
	Tries to use equation of line using their gradient	M1	
	Correct answer only	W1	11

9 **(a)** Rewrites as $1 \times \ln x$ — M1

Evidence of trying to use integration by parts — M1

Obtains $x\ln x$ — W1

Obtains $\int x\left(\frac{1}{x}\right) dx$ – do not penalise if dx omitted — W1

Correct answer – do not penalise if c omitted — MW1

Notes

[A] If set $u = 1$, $\frac{dv}{dx} = \ln x$ then can award M1 only for recognising the need to use parts

(b) Tries to differentiate x with respect to θ — M1

Correct derivative — W1

Tries to change limits — M1

Correct limits — W1

Tries to substitute — M1

Obtains $\sqrt{4 - 4\sin^2\theta}$ — W1

Obtains $\frac{2}{3}\cos\theta\, d\theta$ — W1

Simplifies to $\int \frac{4}{3}\cos^2\theta\, d\theta$ — W1

Tries to use $\cos 2\theta$ — M1

Correctly substituted — W1

Correct integration (cao) — MW1

Correct answer (cao) — MW1

Notes

[A] Error in $\frac{dx}{d\theta}$: M1 W0

May gain: M1 W1 – for limits; M1 – tries to substitute; W1 – $\sqrt{4 - 4\sin^2\theta}$ obtained; M1(ft) – if they try to use $\cos 2\theta$; W1(ft) – if correctly substituted; Max of 7/12

[B] If limits are not initially changed but answer converted back to x and calculated correctly at the end then all marks can be awarded

[C] If limits are not initially changed and answer **not** converted back to x at the end then M0 W0 – for limits and MW0 for final answer i.e. max 9/12 — 17

				AVAILABLE MARKS
10	**(i)**	Evidence of trying to use product rule	M1	
		[1] for each of 2 correct terms	W2	
		Sets their derivative = 0	M1	
		Knows $e^{-x} \neq 0$	MW1	
		Sets $\sin x + \cos x = 0$	M1	
		Obtains $\tan x = -1$	MW1	
		Finds four x values – must be in radian format	W1	

Notes

[A] Error in use of product rule
M1
W0 or W1
M1 possible
MW1 possible
M1 – ft dependent on award of both of previous M1 marks
MW0 and W0
Max of 5/8

	(ii)	Tries to find y values (can ft their values from (i))	M1	
		Four correct values	W1	
		Tries to find r (can ft their values from (i))	M1	
		Correct answer (must try a 2nd ratio)	W1	
	(iii)	Tries formula for sum to infinity (can ft their a, r)	M1	
		Correct answer (either form)	W1	
11	**(i)**	Evidence of trying to find $\frac{dy}{dx}$ using parametric differentiation	M1	
		One mark for each of correct numerator and denominator	MW2	
		Sets their $\frac{dy}{dx} = 0$ (conditional on award of 1st M1)	M1	
		Correct value of t	W1	
		Correct coordinates	MW1	

Notes

[A] Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$ but no further use of parametric differentiation gives max 2/6

	(ii)	Tries to find 2nd derivative	M1	
		Tries to use parametric differentiation	M1	
		Tries to use correct quotient rule	M1	
		One mark for each term differentiated correctly	W2	
		Tests that 2nd derivative is negative → maximum	MW1	12

Notes

[A] Incorrect $\frac{dy}{dx}$ from (i):
Can award any of 1st three M1 marks as appropriate
W1 possible if one of terms is identical to that on MS
MW0
Max of 4/6

			AVAILABLE MARKS
12	Sets up differential equation (allow $\frac{dV}{dt} = k\sqrt{V}$)	M1	
	Tries to separate variables	M1	
	Correctly	W1	
	Tries to integrate	M1	
	Both sides are correct	W1	
	Finds c correctly	MW1	
	Uses $\frac{4}{9}$	M1	
	Finds k	W1	
	Uses $V = 0$	M1	
	Finds correct time	W1	10

Notes

[A] Incorrect initial differential equation e.g. $\frac{dV}{dt} = kV$ or $\frac{dV}{dt} = kV^2$

Award M0
M1 W1 as ft
M1 W0
MW0
M1 W0
M1 W0
Max of 5/10