New Specification

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ADVANCED
General Certificate of Education
2019

Centre Number

Candidate Number

Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

AMT11

[AMT11]
TUESDAY 28 MAY, MORNING

TIME

2 hours 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all twelve** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an H.B. pencil.
Show clearly the full development of your answers. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

1 A curve is given by the equation

$$x^3 + 3y^2 = 11$$

By using implicit differentiation find $\frac{dy}{dx}$ in terms of x and y. [4]

2 A curve is defined parametrically by

$$x = at^2 \quad \text{and} \quad y = 3at$$

where a is a constant and t is the parameter.

Find the Cartesian equation of this curve. [4]

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3 A mirror ABCDE is designed in the form of a sector of a circle, centred at B, together with two congruent right-angled triangles, BAE and BCD, as shown in **Fig. 1** below.

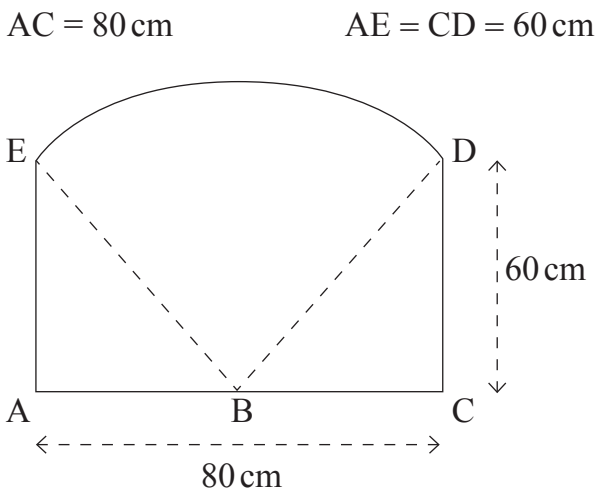

Fig. 1

(i) Find the angle EBD in radians. [5]

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(ii) Find the area of the mirror. [7]

4 **(i)** Prove that

$$\text{cosec } 2\theta - \cot 2\theta \equiv \tan \theta$$ [7]

(ii) Hence find the exact value of $\tan \frac{\pi}{8}$ [2]

[Turn over

5 **(a)** A function f is defined by

$$\text{f}: x \to x^2 - 8, \qquad x \in \mathbb{R}, \qquad x \geqslant 0$$

(i) State the range of the function f(x). [1]

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(ii) Find the inverse function $\text{f}^{-1}(x)$, clearly stating its domain. [4]

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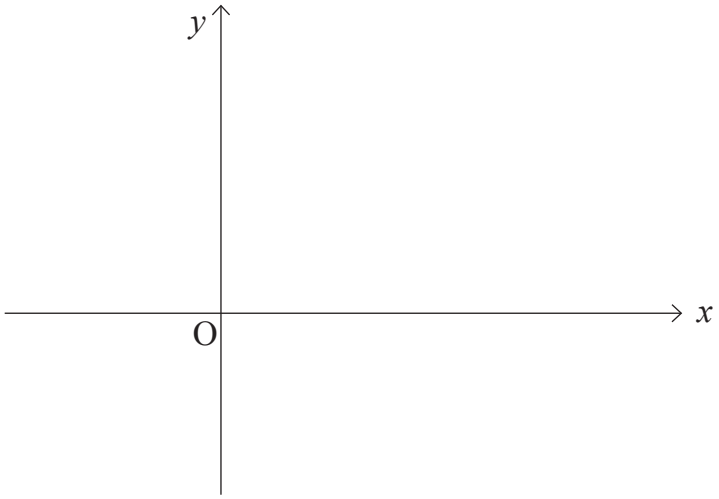
A function g is defined by

$$g: x \to |x - 3|, \quad x \in \mathbb{R}$$

(iii) On the axes below sketch the graph of $y = g(x)$. [2]



(iv) Find the composite function $gf(x)$. [2]

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[Turn over

(b) The graph of the function $y = \mathrm{h}(x)$ is sketched in **Fig. 2** below.

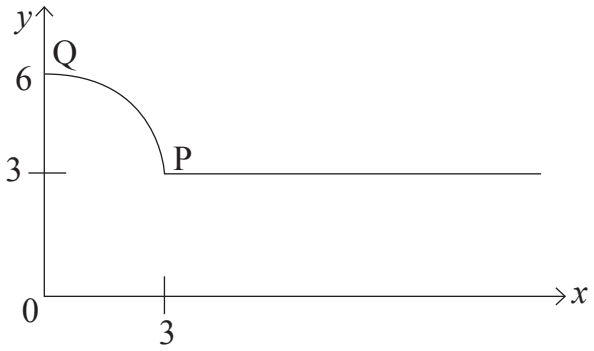
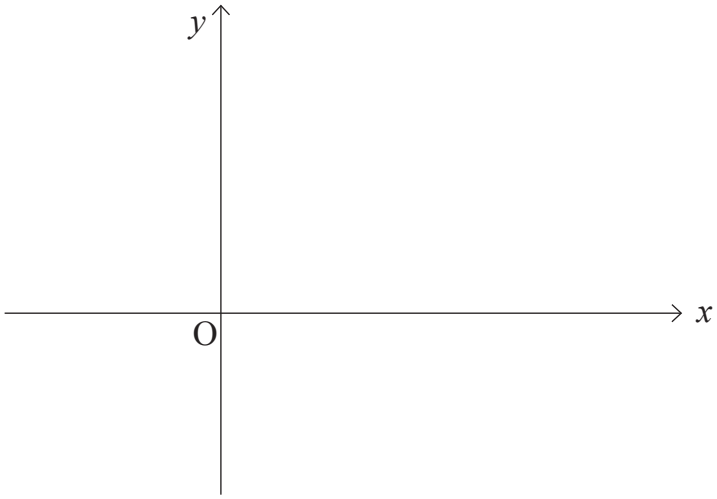


Fig. 2

(i) On the axes below sketch the graph of

$$y = \frac{1}{3}\mathrm{h}(3x)$$

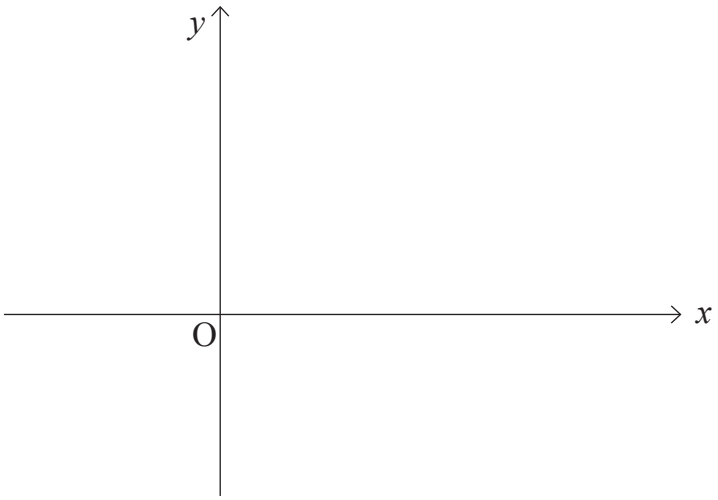
and clearly label the images of the points P and Q. [2]



(ii) On the axes below sketch the graph of

$$y = 6 - \mathrm{h}(x)$$

and clearly label the images of the points P and Q. [2]



[Turn over

6 The expression $8 \sin x + 15 \cos x$ can be written in the form $R \sin(x + \alpha)$, where R is an integer and $0° \leqslant \alpha \leqslant 90°$

(i) Find the values of R and α. [6]

(ii) Hence, or otherwise, determine the maximum value of

$$\frac{18}{8 \sin x + 15 \cos x + 23}$$

and find a corresponding value of x. [4]

7 **(i)** Use the Trapezium Rule with 3 ordinates to find an approximate value for

$$\int_2^3 \frac{x^2}{(x+3)(x-1)}\,\mathrm{d}x$$ [5]

(ii) Use partial fractions to calculate the value of

$$\int_2^3 \frac{x^2}{(x+3)(x-1)}\,\mathrm{d}x \qquad [12]$$

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(iii) Explain how the use of the Trapezium Rule in **(i)** could be modified to obtain a better approximation to the integral

$$\int_2^3 \frac{x^2}{(x+3)(x-1)}\,\mathrm{d}x \qquad [1]$$

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[Turn over

8 The population, P, in a housing development grows at a rate proportional to the population at any time t (years).

This can be modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP$$

where k is a constant.

The initial population is P_0

(i) Show that

$$P = P_0 \mathrm{e}^{kt}$$ [6]

(ii) Given that the initial population doubles in 5 years, find the exact value of k. [3]

[Turn over

(iii) Find the number of years until the initial population is trebled.

Give the answer to the nearest year. [3]

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(iv) State a limitation of this model. [1]

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[Turn over

9 A curve has the equation

$$y = (x - 5) \ln x$$

(i) Show that

$$\frac{dy}{dx} = 1 - \frac{5}{x} + \ln x \qquad [4]$$

(ii) Show that the curve has a turning point between $x = 2$ and $x = 3$ [3]

(iii) By taking 2.4 as a first approximation to the x-coordinate of the turning point, use the Newton Raphson method once to find a better approximation. [5]

10 (a) Find

$$\int x^{-\frac{1}{2}} \ln x \, dx \qquad [7]$$

(b) Using the substitution $u^2 = x^2 + 4$, or otherwise, find the exact value of

$$\int_0^{\sqrt{5}} \frac{x^3}{\sqrt{x^2+4}}\,\mathrm{d}x$$

[8]

11 The graphs of the curves

$$y = \sin 2x \text{ and } y = \cos 2x$$

are shown in **Fig. 3** below.

The curves intersect at the points A and B.

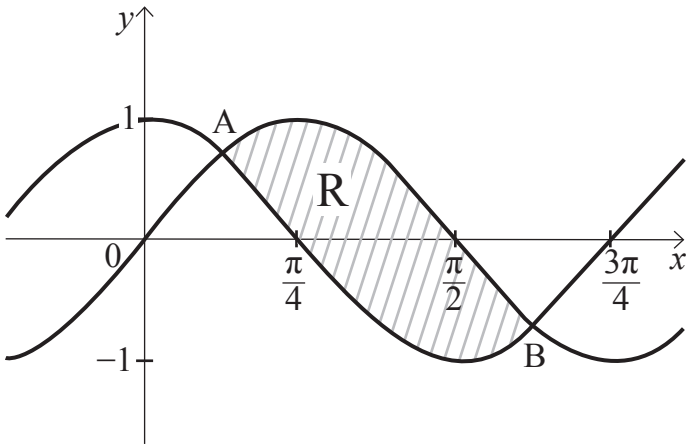

Fig. 3

(i) Show that the x-coordinates of A and B are $\frac{\pi}{8}$ and $\frac{5\pi}{8}$ [4]

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The top section of a trophy is a flat metal sheet modelled in the shape of the shaded region R.

(ii) Calculate the area of this region. [6]

A circle has the equation

$$x^2 + y^2 = 4$$

The base of the trophy can be modelled as the solid formed when the area bounded by this circle, the y-axis and the line $x = 1$ is rotated through 2π radians about the x-axis.

(iii) Find the exact volume of the trophy base. [6]

12 (a) (i) Prove that the sum of n terms of an arithmetic progression with first term a and last term l is

$$S_n = \frac{1}{2} n(a + l)$$ [4]

The first term of an arithmetic progression is 7 and the last term is 79
The sum of the progression is 1075

(ii) Find the number of terms. [3]

(iii) Find the common difference. [3]

[Turn over

(b) A salesman receives a bonus at the end of each year and decides to invest this money in a savings account.

At the end of Year 1 he invests £400

At the end of Year 2 he invests a further £400 and receives 2% interest on the first year's £400

At the end of Year 3 he invests a further £400 and 2% interest is added to the total sum of money which he has accumulated during the first two years.

(i) Show that he has £1,224.16 in his account at the end of Year 3 [4]

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(ii) Assuming that the man continues to invest in this way, form and sum a series to prove that he will have

$$£20\,000(1.02^n - 1)$$

in his account at the end of n years. [6]

[Turn over

(iii) Hence find the least number of years until his investment exceeds £7,000 [4]

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For Examiner's use only	
Question Number	**Marks**
1	
2	
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