Rewarding Learning

ADVANCED
General Certificate of Education
2019

Mathematics

Assessment Unit A2 1

assessing

Pure Mathematics

[AMT11]

TUESDAY 28 MAY, MORNING

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

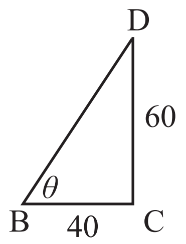
Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

		Marks	AVAILABLE MARKS
1	$x^3 + 3y^2 = 11$		
	Differentiate to give $3x^2 + 6y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	MW3	
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{x^2}{2y}$	MW1	4
2	$t = \dfrac{y}{3a}$	M1 W1	
	$\Rightarrow x = a\left(\dfrac{y^2}{9a^2}\right)$	M1	
	$\Rightarrow y^2 = 9ax$	W1	
	Alternative Solution		
	$y^2 = 9a^2t^2$	M1 W1	
	$\Rightarrow y^2 = 9a(at^2)$	M1	
	$\Rightarrow y^2 = 9ax$	W1	4
3	**(i)** $\tan\theta = \dfrac{60}{40}$	M1 W1	
	$\Rightarrow \theta = 0.98279\ldots$	W1	

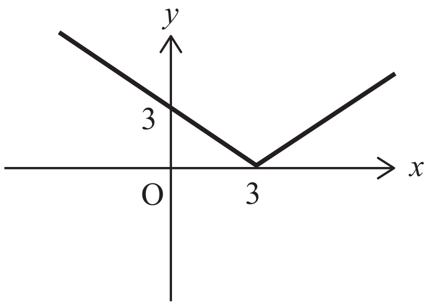


		Marks	AVAILABLE MARKS
	$\Rightarrow \angle\text{EBD} = \pi - 2 \times 0.98279$	M1	
	$= 1.17600\ldots$		
	$= 1.18$ radians	W1	
	(ii) $\text{BD} = \sqrt{60^2 + 40^2}$	M1	
	$= 20\sqrt{13}$	W1	
	Area of sector $= \dfrac{1}{2} \times (20\sqrt{13})^2 \times 1.17600..$	M1	
	$= 3057.6\ldots$	W1	
	Area of 2 triangles $= 2 \times \dfrac{1}{2} \times 60 \times 40$	M1	
	$= 2400$	W1	
	Total area $= 5457.6\,\text{cm}^2$	MW1	
	$= 5460\ \text{cm}^2$ (3sf)		12

			AVAILABLE MARKS
4 **(i)**	$\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan\theta$		
	$\Rightarrow \dfrac{1}{\sin 2\theta} - \dfrac{\cos 2\theta}{\sin 2\theta}$	MW2	
	$\Rightarrow \dfrac{1 - \cos 2\theta}{\sin 2\theta}$	MW1	
	$\Rightarrow \dfrac{1 - (1 - 2\sin^2\theta)}{\sin 2\theta}$	MW1	
	$\Rightarrow \dfrac{2\sin^2\theta}{2\sin\theta\cos\theta}$	W1 MW1	
	$\Rightarrow \tan\theta$	W1	
(ii)	$\tan\dfrac{\pi}{8} = \operatorname{cosec}\dfrac{\pi}{4} - \cot\dfrac{\pi}{4}$	M1	
	$\Rightarrow \sqrt{2} - 1$	W1	9

					AVAILABLE MARKS
5	(a)	(i)	$f(x) \geqslant -8$	MW1	
		(ii)	$y = x^2 - 8$	M1	
			$\Rightarrow x^2 = y + 8$	MW1	
			$\Rightarrow x = \sqrt{y + 8}$	MW1	
			$\Rightarrow f^{-1} : x \rightarrow \sqrt{x + 8}, x \in \mathbb{R}, x \geqslant -8$	MW1	

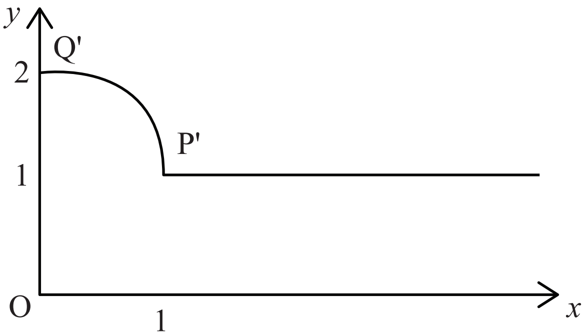
(iii)



MW1 W1

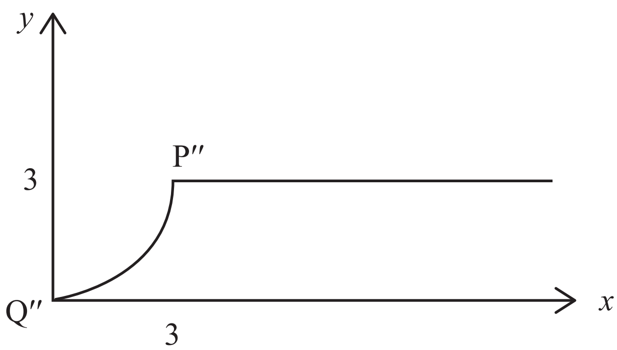
(iv)	gf: $x \rightarrow \lvert (x^2 - 8) - 3 \rvert$	M1	
	$\Rightarrow$ gf: $x \rightarrow \lvert x^2 - 11 \rvert, x \in \mathbb{R}, x \geqslant 0$	W1	

(b) (i)



MW1
W1

(ii)



MW1
W1

13

			AVAILABLE MARKS
6	**(i)** $8 \sin x + 15 \cos x \equiv R(\sin x \cos \alpha + \cos x \sin \alpha)$	MW1	
	$\Rightarrow R \cos \alpha = 8$ and $R \sin \alpha = 15$	M1	
	$\Rightarrow \tan \alpha = \frac{15}{8}$	M1	
	$\Rightarrow \alpha = 61.9°$	W1	
	Also $R = \sqrt{8^2 + 15^2}$	M1	
	$\Rightarrow R = 17$	W1	
	(ii) Function can be re-written as $\frac{18}{17 \sin (x + \alpha) + 23}$	M1	
	Maximum occurs when denominator is minimum.		
	This occurs when $\sin (x + \alpha) = -1$	M1	
	Maximum value $= \frac{18}{-17 + 23}$		
	$= 3$	W1	
	$\Rightarrow x + \alpha = 270°$ $\Rightarrow x = 208°$	MW1	
	(Alternative answer $x = -152°$)		10

		AVAILABLE MARKS

7 **(i)**

x	$\frac{x^2}{(x+3)(x-1)}$
2	$\frac{4}{5} = 0.8$
2.5	$\frac{25}{33} = 0.7575\ ...$
3	$\frac{3}{4} = 0.75$

MW1

MW1

W1

$\Rightarrow \frac{1}{2} \times 0.5 \times \{0.8 + 0.75 + 2(0.7575)\ ...\}$ M1

$= 0.766$ W1

(ii)

$$\begin{array}{r|l} & 1 \\ \hline x^2+2x-3 & x^2 \\ & \underline{x^2+2x-3} \\ & -2x+3 \end{array}$$

M1

W1

$\Rightarrow \frac{x^2}{(x+3)(x-1)} = 1 + \frac{-2x+3}{(x+3)(x-1)}$ W1

$\frac{-2x+3}{(x+3)(x-1)} \equiv \frac{A}{x+3} + \frac{B}{x-1}$ M1

$\Rightarrow -2x + 3 \equiv A(x-1) + B(x+3)$ MW1

Let $x = 1 \Rightarrow 1 = 4B$ M1

$\Rightarrow B = \frac{1}{4}$ W1

Let $x = -3 \Rightarrow 9 = -4A$

$\Rightarrow A = \frac{-9}{4}$ MW1

$\Rightarrow \int_2^3 \left(1 - \frac{\frac{9}{4}}{x+3} + \frac{\frac{1}{4}}{x-1}\right) dx$

$= [x - \frac{9}{4} \ln |x+3| + \frac{1}{4} \ln |x-1|]_2^3$ MW3

$= [3 - \frac{9}{4} \ln 6 + \frac{1}{4} \ln 2] - [2 - \frac{9}{4} \ln 5 + \frac{1}{4} \ln 1]$

$= 1 - \frac{9}{4} \ln 6 + \frac{9}{4} \ln 5 + \frac{1}{4} \ln 2$ W1

$= 0.763$ (3 sf)

(iii) Use more strips (or smaller intervals) to improve the approximation MW1 18

			AVAILABLE MARKS
8	**(i)**	$\frac{dP}{dt} = kP$	
		$\Rightarrow \int \frac{1}{P}\,dP = \int k\,dt$ — M1 W1	
		$\Rightarrow \ln P = kt + c$ — MW1	
		When $t = 0$, $P = P_0$ — M1	
		$\Rightarrow \ln P_0 = c$ — W1	
		$\Rightarrow \ln P = kt + \ln P_0$ $\Rightarrow \ln \frac{P}{P_0} = kt$ $\Rightarrow P = P_0 e^{kt}$ — MW1	
	(ii)	$P = P_0 e^{kt}$ $\Rightarrow 2P_0 = P_0 e^{5k}$ — M1	
		$\Rightarrow e^{5k} = 2$ — W1	
		$\Rightarrow k = \frac{1}{5}\ln 2$ — W1	
		Alternative solution When $t = 5$, $P = 2P_0$ — M1	
		$\Rightarrow \ln(2P_0) = 5k + \ln P_0$ — W1 $\Rightarrow \ln 2 = 5k$ $\Rightarrow k = \frac{1}{5}\ln 2$ — W1	
	(iii)	When $P = 3P_0$ — M1	
		$\Rightarrow 3P_0 = P_0 e^{(\frac{1}{5}\ln 2)t}$ — MW1	
		$\Rightarrow e^{(\frac{1}{5}\ln 2)t} = 3$ $\Rightarrow t = \frac{\ln 3}{\frac{1}{5}\ln 2}$ $\Rightarrow t = 7.92$	
		Time is 8 years — W1	
		Alternative solution When $P = 3P_0$ — M1	
		$\Rightarrow \ln(3P_0) = \left(\frac{1}{5}\ln 2\right)t + \ln P_0$ — MW1	
		$\Rightarrow \frac{\ln 3}{\frac{1}{5}\ln 2} = t$	
		$\Rightarrow t = 7.92$ Time is 8 years — W1	
	(iv)	Population cannot grow indefinitely since the number of people who live in each house (and the number of houses) is finite. — MW1	13

		AVAILABLE MARKS
9 **(i)** $y = (x - 5)\ln x$		
$\Rightarrow \frac{dy}{dx} = (x-5)\frac{1}{x} + \ln x$	M1 W2	
$\Rightarrow \frac{dy}{dx} = 1 - \frac{5}{x} + \ln x$	MW1	
(ii) $f(x) = 1 - \frac{5}{x} + \ln x$		
$f(2) = -0.80685\ldots$	M1	
$f(3) = 0.43194\ldots$	W1	
Since the gradient of $y = (x-5)\ln x$ has a change of sign between $x = 2$ and $x = 3$, and is continuous in this region, then the curve has a turning point between $x = 2$ and $x = 3$	MW1	
(iii) $f(x) = 1 - \frac{5}{x} + \ln x$		
$\Rightarrow f'(x) = \frac{5}{x^2} + \frac{1}{x}$	M1 W1	
$\Rightarrow x_1 = 2.4 - \frac{\left(1 - \frac{5}{2.4} + \ln 2.4\right)}{\frac{5}{2.4^2} + \frac{1}{2.4}}$	M1 W1	
$x_1 = 2.56$ (3 sf)	MW1	12

				AVAILABLE MARKS
10 **(a)** $\int x^{-\frac{1}{2}} \ln x \, dx$	$u = \ln x$	$\frac{dv}{dx} = x^{-\frac{1}{2}}$	M1 W1	
	$\frac{du}{dx} = \frac{1}{x}$	$v = 2x^{\frac{1}{2}}$	MW2	
$\Rightarrow 2x^{\frac{1}{2}} \ln x - \int \frac{1}{x} \times 2x^{\frac{1}{2}} \, dx$			MW2	
$\Rightarrow 2x^{\frac{1}{2}} \ln x - \int 2x^{-\frac{1}{2}} \, dx$				
$\Rightarrow 2x^{\frac{1}{2}} \ln x - 4x^{\frac{1}{2}} + c$			MW1	
(b) $\int_0^{\sqrt{5}} \frac{x^3}{\sqrt{x^2+4}} \, dx$	$u^2 = x^2 + 4$			
	$\Rightarrow 2u\,du = 2x\,dx$		M1	
	$\Rightarrow dx = \frac{u}{x} du$		W1	
	$x = \sqrt{5} \quad \Rightarrow u = 3$		MW1	
	$x = 0 \quad \Rightarrow u = 2$			
$\Rightarrow \int_2^3 \frac{(u^2-4)^{\frac{3}{2}}}{u} \times \frac{u}{(u^2-4)^{\frac{1}{2}}} \, du$			M1 W1	
$\Rightarrow \int_2^3 (u^2-4)\,du$			MW1	
$\Rightarrow \left[\frac{1}{3}u^3 - 4u\right]_2^3$			MW1	
$\Rightarrow \left[9 - 12\right] - \left[\frac{8}{3} - 8\right]$				
$= 2\frac{1}{3}$			MW1	15

AVAILABLE MARKS

11 **(i)** $\sin 2x = \cos 2x$ M1

$\Rightarrow \tan 2x = 1$ MW1

$\Rightarrow 2x = \frac{\pi}{4}, \frac{5\pi}{4}$ W1

$\Rightarrow x = \frac{\pi}{8}, \frac{5\pi}{8}$ W1

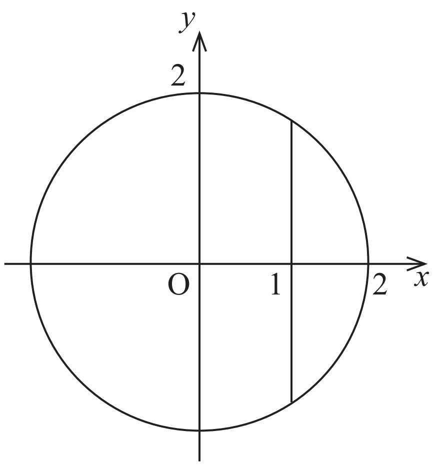
(ii) Hence area is given by $\int_{\frac{\pi}{8}}^{\frac{5\pi}{8}} (\sin 2x - \cos 2x)\, dx$ M1 W1 MW1

$= \left[-\frac{1}{2}\cos 2x - \frac{1}{2}\sin 2x\right]_{\frac{\pi}{8}}^{\frac{5\pi}{8}}$ MW2

$= \left[\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right] - \left[-\frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right]$

$= \sqrt{2}$ MW1

(iii)



$V = \pi \int y^2 dx$ M1

$= \pi \int_0^1 (4 - x^2) dx$ W2

$= \pi \left[4x - \frac{1}{3}x^3\right]_0^1$ MW2

$= \pi \left[4 - \frac{1}{3}\right]$

$= \frac{11\pi}{3}$ MW1 16

12 (a) (i) $S_n = a + (a + d) + \ldots \quad \ldots + (l - d) + l$	M1	**AVAILABLE MARKS**
Also $S_n = l + (l - d) + \ldots \quad \ldots + (a + d) + a$		
$\Rightarrow 2S_n = (a + l) + (a + l) + \ldots \quad \ldots + (a + l) + (a + l)$	M1W1	
$\Rightarrow 2S_n = n(a + l)$		
$\Rightarrow \quad S_n = \frac{1}{2}n(a + l)$	MW1	
(ii) $\frac{1}{2}n(7 + 79) = 1075$	M1 W1	
$\Rightarrow \quad 86n = 2150$		
$\Rightarrow \quad n = 25$	MW1	
(iii) $7 + 24d = 79$	M1 W1	
$\Rightarrow \; 24d = 72$		
$\Rightarrow \quad d = 3$	MW1	
(b) (i) Year 1: 400		
Year 2: $400 \times 1.02 + 400$	M1 W1	
Year 3: $400 \times 1.02^2 + 400 \times 1.02 + 400$	MW1	
Hence he has £1,224.16	W1	
(ii) Year n: $400 \times 1.02^{n-1} + 400 \times 1.02^{n-2} + \ldots + 400 \times 1.02 + 400$	MW1	
This is a GP with $a = 400$, $r = 1.02$	W1 W1	
$\Rightarrow S_n = \frac{400(1.02^n - 1)}{1.02 - 1}$	M1 W1	
$\Rightarrow S_n = 20\,000\,(1.02^n - 1)$	MW1	
(iii) $20\,000\,(1.02^n - 1) > 7\,000$	M1	
$\Rightarrow 1.02^n - 1 > 0.35$		
$\Rightarrow 1.02^n > 1.35$	W1	
$\Rightarrow n \ln 1.02 > \ln 1.35$	M1	
$\Rightarrow n > 15.2$		
Hence it will take 16 years to exceed £7,000	W1	24
	Total	**150**