



*Rewarding Learning*

**ADVANCED SUBSIDIARY (AS)  
General Certificate of Education  
2017**

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**Mathematics**

Assessment Unit F1

*assessing*

Module FP1: Further Pure Mathematics 1

**[AMF11]**

**TUESDAY 13 JUNE, MORNING**

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**MARK  
SCHEME**

## GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

### Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

**M** indicates marks for correct method.

**W** indicates marks for working.

**MW** indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

### Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

- (a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
- (b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1 (i)  $(p + q\sqrt{3})(r + s\sqrt{3})$  M1

$$= pr + qr\sqrt{3} + ps\sqrt{3} + 3qs$$

W1

$$= (pr + 3qs) + (qr + ps)\sqrt{3}$$

Since this number is of the same format as the original numbers,  
then S is closed. MW1

(ii)  $(p + q\sqrt{3})(f) = 1$  M1

$$\Rightarrow f = \frac{1}{p + q\sqrt{3}} \times \frac{p - q\sqrt{3}}{p - q\sqrt{3}}$$

MW1

$$\Rightarrow f = \frac{p - q\sqrt{3}}{p^2 - 3q^2}$$

$$\Rightarrow f = \frac{p}{p^2 - 3q^2} + \left(\frac{-q}{p^2 - 3q^2}\right)\sqrt{3}$$

MW1

where  $p^2 \neq 3q^2$

AVAILABLE  
MARKS

6

2 (i)  $\det(\mathbf{A} - \lambda \mathbf{1}) = 0$

M1

$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 & -2 \\ 0 & -3-\lambda & 0 \\ 2 & 0 & -2-\lambda \end{vmatrix} = 0$$

MW1

$$\Rightarrow (1-\lambda)\{(-3-\lambda)(-2-\lambda)\} - 1(0) - 2\{-2(-3-\lambda)\} = 0$$

MW1

$$\Rightarrow (-3-\lambda)\{(1-\lambda)(-2-\lambda) + 4\} = 0$$

$$\Rightarrow (3+\lambda)(-2+2\lambda-\lambda+\lambda^2+4) = 0$$

$$\Rightarrow (3+\lambda)(\lambda^2+\lambda+2) = 0$$

W1

$$\Rightarrow \lambda = -3 \quad \text{or} \quad \lambda^2 + \lambda + 2 = 0$$

$$\Rightarrow \lambda = \frac{-1 \pm \sqrt{1-8}}{2}$$

$\Rightarrow$  no real solutions

MW1

Hence  $\lambda = -3$  is the only real eigenvalue

W1

(ii)  $\begin{pmatrix} 1 & 1 & -2 \\ 0 & -3 & 0 \\ 2 & 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -3 \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

M1

This gives

MW1

$$\begin{aligned} x + y - 2z &= -3x \\ -3y &= -3y \\ 2x - 2z &= -3z \end{aligned}$$

which simplifies to

$$\begin{aligned} 4x + y - 2z &= 0 \\ 2x + z &= 0 \end{aligned}$$

$$\Rightarrow z = -2x$$

$$\Rightarrow 4x + y + 4x = 0$$

$$\Rightarrow 8x + y = 0$$

$$\Rightarrow y = -8x$$

$$\Rightarrow \text{a possible eigenvalue is } \begin{pmatrix} x \\ -8x \\ -2x \end{pmatrix}$$

MW1

and a corresponding unit eigenvector is  $\frac{1}{\sqrt{69}} \begin{pmatrix} 1 \\ -8 \\ -2 \end{pmatrix}$

W1

AVAILABLE  
MARKS

10

3 (i)  $C_1 : (x + 1)^2 + (y - 4)^2 = 13$

Hence centre is  $(-1, 4)$  and radius  $= \sqrt{13}$

MW2

$C_2 : (x - 5)^2 + (y - 13)^2 = 52$

Hence centre is  $(5, 13)$  and radius  $= 2\sqrt{13}$

MW2

If circles touch externally then distance between centres must equal the sum of the radii.

M1

Distance between centres  $= \sqrt{6^2 + 9^2}$

$= \sqrt{117}$

MW1

$= 3\sqrt{13}$

Since  $3\sqrt{13} = \sqrt{13} + 2\sqrt{13}$ , then the circles touch externally.

MW1

(ii) Substitute to give  $x^2 + (3x + k)^2 + 2x - 8(3x + k) + 4 = 0$

MW1

$\Rightarrow x^2 + 9x^2 + 6xk + k^2 + 2x - 24x - 8k + 4 = 0$

W1

$\Rightarrow 10x^2 + x(6k - 22) + (k^2 - 8k + 4) = 0$

For a tangent to exist, then  $b^2 - 4ac = 0$

M1

$\Rightarrow (6k - 22)^2 - 4(10)(k^2 - 8k + 4) = 0$

W1

$\Rightarrow 36k^2 - 264k + 484 - 40k^2 + 320k - 160 = 0$

$\Rightarrow -4k^2 + 56k + 324 = 0$

$\Rightarrow k^2 - 14k - 81 = 0$

MW1

$\Rightarrow k = \frac{14 \pm \sqrt{14^2 + 4(81)}}{2}$

$\Rightarrow k = \frac{14 \pm 2\sqrt{130}}{2}$

$\Rightarrow k = 7 \pm \sqrt{130}$

MW1

AVAILABLE  
MARKS

13

		AVAILABLE MARKS
4	(a) Shear, parallel to the $y$ -axis and with scale factor 3	MW3
	(b) (i) $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} X \\ Y \end{pmatrix}$	M1
	$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$	M1 MW1
	$\Rightarrow x = \frac{1}{10} (4X - 2Y)$	W1
	and $y = \frac{1}{10} (-X + 3Y)$	
	Substitute into the equation of the circle to give	M1
	$\frac{1}{100} (4X - 2Y)^2 + \frac{1}{100} (-X + 3Y)^2 = 9$	W1
	$\Rightarrow 16X^2 - 16XY + 4Y^2 + X^2 - 6XY + 9Y^2 = 900$	MW1
	$\Rightarrow 17X^2 + 13Y^2 - 22XY - 900 = 0$	W1
	(ii) Area of original circle = $\pi(3)^2 = 9\pi$	MW1
	Area of new region = $\det \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \times 9\pi$	M1
	$= 10 \times 9\pi$	
	$= 90\pi$ square units	MW1
		14

5 (i)  $\det \mathbf{N} = 0$

M1

$$\Rightarrow \begin{vmatrix} 0 & -1 & 2 \\ 1 & 1 & 2 \\ -1 & p & 1 \end{vmatrix} = 0$$

MW1

$$\Rightarrow 1(1+2) + 2(p+1) = 0$$

MW1

$$\Rightarrow 3 + 2p + 2 = 0$$

$$\Rightarrow 2p + 5 = 0$$

$$\Rightarrow p = -2\frac{1}{2}$$

W1

(ii) 
$$\begin{vmatrix} 0 & -1 & 2 \\ 1 & 1 & 2 \\ -1 & 3 & 1 \end{vmatrix} = 1(1+2) + 2(3+1) = 11$$

MW1

Matrix of minors = 
$$\begin{pmatrix} -5 & 3 & 4 \\ -7 & 2 & -1 \\ -4 & -2 & 1 \end{pmatrix}$$

MW3

Matrix of cofactors = 
$$\begin{pmatrix} -5 & -3 & 4 \\ 7 & 2 & 1 \\ -4 & 2 & 1 \end{pmatrix}$$

MW1

Inverse = 
$$\frac{1}{11} \begin{pmatrix} -5 & 7 & -4 \\ -3 & 2 & 2 \\ 4 & 1 & 1 \end{pmatrix}$$

M1 W1

(iii) 
$$\begin{pmatrix} 0 & -1 & 2 \\ 1 & 1 & 2 \\ -1 & 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \\ 19 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 1 & 2 \\ -1 & 3 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -5 \\ 1 \\ 19 \end{pmatrix}$$

M1

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{11} \begin{pmatrix} -5 & 7 & -4 \\ -3 & 2 & 2 \\ 4 & 1 & 1 \end{pmatrix} \begin{pmatrix} -5 \\ 1 \\ 19 \end{pmatrix}$$

MW1

$$= \frac{1}{11} \begin{pmatrix} -44 \\ 55 \\ 0 \end{pmatrix}$$

W1

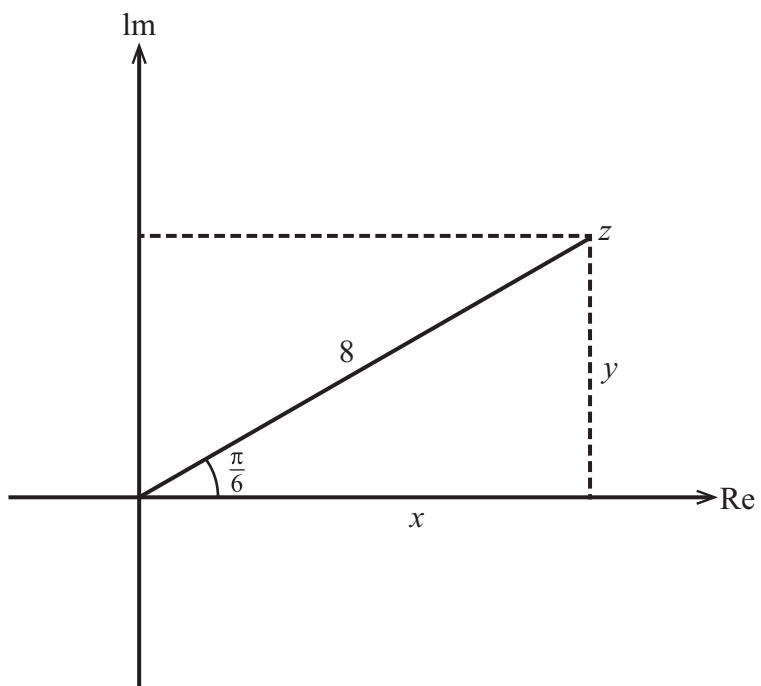
$$\Rightarrow x = -4, y = 5, z = 0$$

W1

AVAILABLE  
MARKS

15

6 (a)



$$x = 8 \cos \frac{\pi}{6} \quad y = 8 \sin \frac{\pi}{6}$$

$$\Rightarrow z = 4\sqrt{3} + 4i$$

AVAILABLE  
MARKS

M1

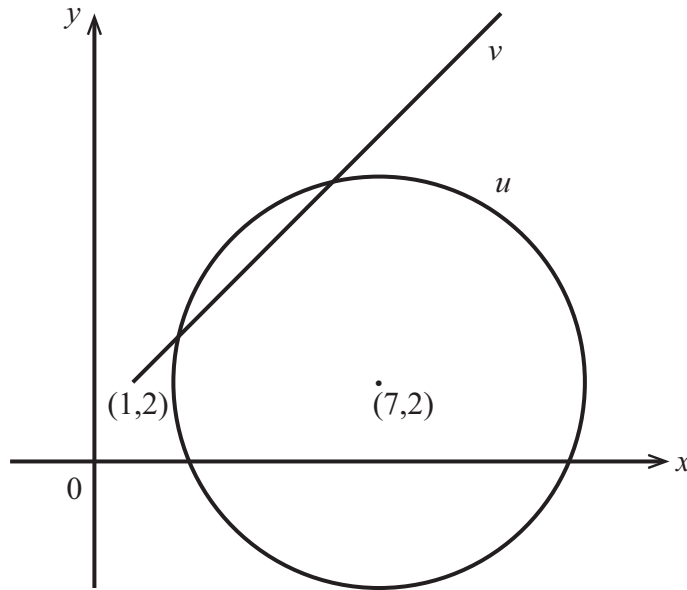
W2

W1



(b) (i) Circle centre (7, 2) with radius  $\sqrt{20}$

MW3



(ii) Half-line through (1,2), at angle of  $\frac{\pi}{4}$  to the x-axis.

MW3

(iii) Equation for  $u$ :  $(x - 7)^2 + (y - 2)^2 = 20$  ①

MW1

Equation for  $v$ :  $y = x + c$  as  $\tan \frac{\pi}{4} = 1$

MW1

Since line passes through (1, 2), then equation is  $y = x + 1$  ②

MW1

Substitute ② into ① to give  $(x - 7)^2 + (x - 1)^2 = 20$

M1

Simplify to give  $2x^2 - 16x + 30 = 0$

$$\Rightarrow x^2 - 8x + 15 = 0$$

W1

$$\Rightarrow (x - 5)(x - 3) = 0$$

$$\Rightarrow x = 5, 3$$

MW1

$$\Rightarrow y = 6, 4$$

Hence points of intersection are (5, 6) and (3, 4)

W1

**Total**

AVAILABLE  
MARKS

17

**75**