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2015

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# Mathematics

Assessment Unit C4  
*assessing*  
Module C4: Core Mathematics 4



[AMC41]  
TUESDAY 26 MAY, MORNING

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## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.  
Answer **all eight** questions.  
Show clearly the full development of your answers.  
Answers should be given to three significant figures unless otherwise stated.  
You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75  
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.  
A copy of the **Mathematical Formulae and Tables booklet** is provided.  
Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

**Answer all eight questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

- 1** Find the angle between the two lines whose vector equations are

$$\mathbf{r}_1 = \mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j})$$

$$\mathbf{r}_2 = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad [5]$$

- 2** Using the substitution  $u = x - 1$  or otherwise find

$$\int_2^5 \frac{x-2}{\sqrt{x-1}} dx \quad [6]$$

- 3 (a)** A curve is given by the parametric equations

$$x = e^{2t} \quad y = 1 + e^t$$

Find the gradient of the curve when  $t = 0$  [5]

- (b)** Find the equation of the normal to the curve

$$2x^2 + y^2 - 3y = 0$$

at the point (1, 2). [9]

- 4 The number of yeast cells increases at a rate proportional to the number of yeast cells,  $N$ , present at any time  $t$ .

This can be modelled by the differential equation

$$\frac{dN}{dt} = 0.02N$$

If the number of yeast cells present at time  $t = 0$  minutes is  $N_0$ , find the length of time it will take for the number of yeast cells to be  $2N_0$  [8]

- 5 (a) Sketch the graph of

$$y = \tan^{-1}x$$

where  $-\frac{\pi}{2} < y < \frac{\pi}{2}$  [2]

- (b) Solve the equation

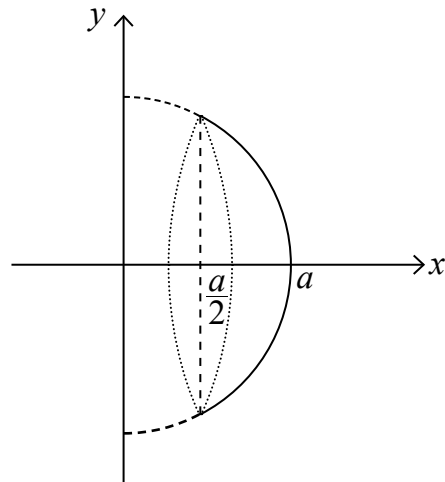
$$\tan 2\theta = 3 \cot \theta$$

where  $0^\circ \leq \theta \leq 360^\circ$  [8]

- 6 The maximum volume of liquid that a chinese wok can hold can be modelled by the volume generated by the rotation of the curve

$$x^2 + y^2 = a^2$$

through  $2\pi$  radians about the  $x$ -axis between  $x = \frac{a}{2}$  and  $x = a$  as shown in **Fig. 1** below.



**Fig. 1**

- (i) Find the volume generated by the rotation. [7]
- (ii) Given that when the wok is ‘full’ it holds  $2880\pi \text{ cm}^3$ , find  $a$ . [2]

7 A function  $f$  is defined by

$$f : x \rightarrow \frac{1}{x+2} \quad x \in \mathbb{R} \ x > -2$$

(i) Sketch the graph of  $y = f(x)$ . [1]

(ii) State the equations of the asymptotes to this graph. [2]

A function  $g$  is defined by

$$g : x \rightarrow \frac{1}{x-1} \quad x \in \mathbb{R} \ x > 3$$

(iii) Find the range of  $g$ . [1]

(iv) Find the composite function  $fg$ . [2]

(v) Find the inverse function  $(fg)^{-1}$  [4]

8 (a) Find

$$\int x^2 \ln x^2 \, dx \quad [6]$$

(b) Find the exact value of

$$\int_0^{\frac{\pi}{4}} \cos^2 x \sin^3 x \, dx \quad [7]$$

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**THIS IS THE END OF THE QUESTION PAPER**

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