Oxford Cambridge and RSA

Tuesday 14 June 2022 – Afternoon

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Time allowed: 2 hours

9121333299

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **16** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)\big(f(x)\big)^n\,dx = \frac{1}{n+1}\big(f(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

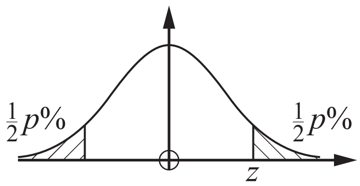
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (23 marks)

1 Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are exact values to be determined. **[4]**

2 Find the sum of the infinite series 50 + 25 + 12.5 + 6.25 + … . **[2]**

3 **(a)** On the axes in the Printed Answer Booklet, sketch the curve with equation $y = 3 \times 0.4^x$. **[3]**

(b) Given that $3 \times 0.4^x = 0.8$, determine the value of x correct to **3** significant figures. **[3]**

4 A survey of university students revealed that

- 31% have a part-time job but do not play competitive sport.
- 23% play competitive sport but do not have a part-time job.
- 22% do not play competitive sport and do not have a part-time job.

(a) Show this information on a Venn diagram. **[2]**

A student is selected at random.

(b) Determine the probability that the student plays competitive sport **and** has a part-time job. **[2]**

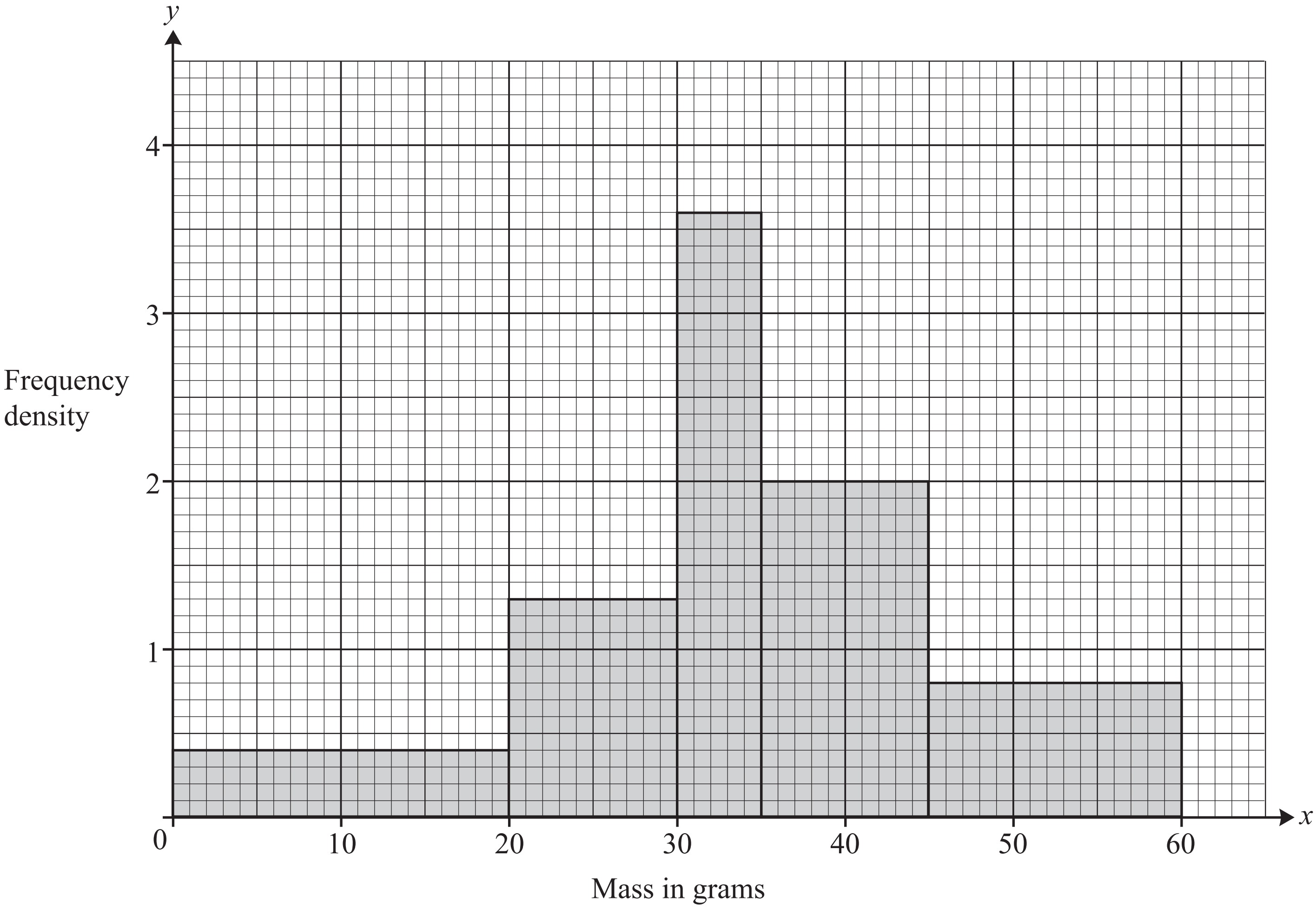
5 Tom conjectures that if n is an **odd** number greater than 1, then $2^n - 1$ is prime.

Find a counter example to disprove Tom's conjecture. **[3]**

6 X is a continuous random variable such that $X \sim \mathrm{N}(\mu, \sigma^2)$.

On the sketch of this Normal distribution in the Printed Answer Booklet, shade the area bounded by the curve, the x-axis and the lines $x = \mu \pm \sigma$. **[2]**

7 Kareem bought some tomatoes. He recorded the mass of each tomato and displayed the results in a histogram, which is shown below.

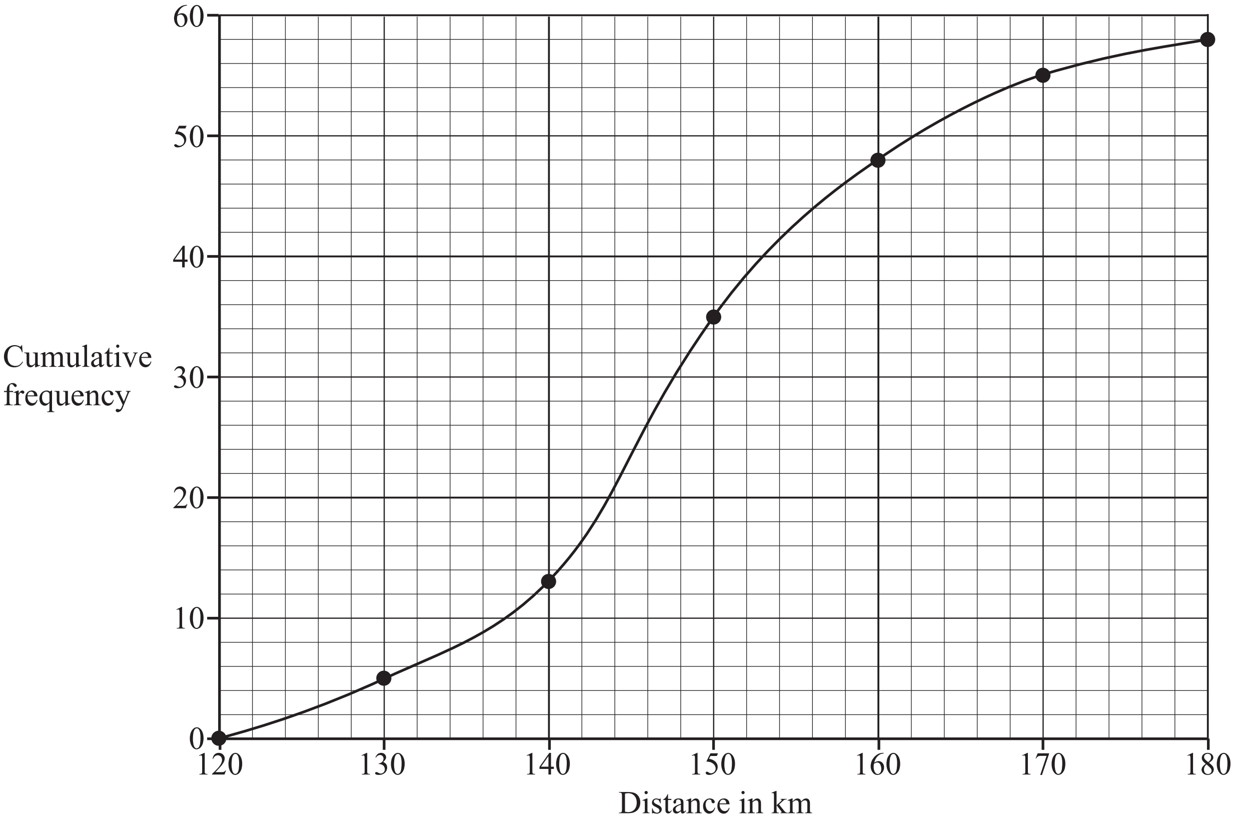


Determine how many tomatoes Kareem bought. **[2]**

Answer **all** the questions.

Section B (77 marks)

8 Ali conducted an investigation into the distances ridden by those members of a cycling club who rode at least 120 km in a training week. She grouped **all** the distances into intervals of length 10 km and then constructed a cumulative frequency diagram, which is shown below.



(a) Explain whether the data Ali used is a sample or a population. **[1]**

The club is taking part in a competition. Eight team members and one reserve are to be selected. The club captain decides that the team members should be those cyclists who rode the furthest during the training week, and that the reserve should be the cyclist who rode the next furthest.

(b) Use the graph to estimate the shortest distance cycled by a team member. **[1]**

The captain's best friend rode 156 km in the training week and was selected as reserve. Ali complained that this was unjustifiable.

(c) Explain whether there is sufficient evidence in the diagram to support Ali's complaint. **[1]**

9 At the beginning of the academic year, all the pupils in year 12 at a college take part in an assessment. Summary statistics for the marks obtained by the 2021 cohort are given below.

$n = 205 \quad \sum x = 23\,042 \quad \sum x^2 = 2\,591\,716$

Marks may only be whole numbers, but the Head of Mathematics believes that the distribution of marks may be modelled by a Normal distribution.

(a) Calculate

- The mean mark
- The variance of the marks **[2]**

(b) Use your answers to part **(a)** to write down a possible Normal model for the distribution of marks. **[2]**

One candidate in the cohort scored less than 105.

(c) Determine whether the model found in part **(b)** is consistent with this information. **[3]**

(d) Use the model to calculate an estimate of the number of candidates who scored 115 marks. **[2]**

10 The parametric equations of a curve are

$x = 2 + 5\cos\theta$ and $y = 1 + 5\sin\theta$, where $0 \leqslant \theta \leqslant 2\pi$.

(a) Determine the cartesian equation of the curve. **[3]**

(b) Hence or otherwise, find the equation of the tangent to the curve at the point $(5, -3)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be determined. **[4]**

11 A die in the form of a dodecahedron has its faces numbered from 1 to 12. The die is biased so that the probability that a score of 12 is achieved is different from any other score. The probability distribution of X, the score on the die, is given in the table in terms of p and k, where $0 < p < 1$ and k is a positive integer.

x	1	2	3	4	5	6	7	8	9	10	11	12
$\mathrm{P}(X = x)$	p	p	p	p	p	p	p	p	p	p	p	kp

Sam rolls the die 30 times, Leo rolls the die 60 times and Nina rolls the die 120 times. They each plot their scores on bar line graphs.

(a) Explain whose graph is most likely to give the best representation of the theoretical probability distribution for the score on the die. **[1]**

(b) Find p in terms of k. **[2]**

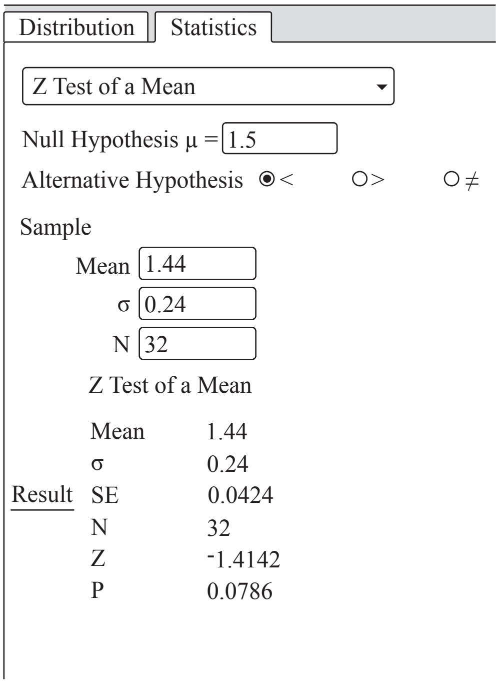
(c) Determine, in terms of k, the expected number of times Nina rolls a 12. **[3]**

(d) Given that Nina rolls a 12 on 32 occasions, calculate an estimate of the value of k. **[2]**

Nina rolls the die a further 30 times.

(e) Use your answer to part **(d)** to calculate an estimate for the probability that she obtains a 12 exactly 8 times in these 30 rolls. **[2]**

12 A retailer sells bags of flour which are advertised as containing 1.5 kg of flour. A trading standards officer is investigating whether there is enough flour in each bag. He collects a random sample and uses software to carry out a hypothesis test at the 5% level. The analysis is shown in the software printout below.



(a) State the hypotheses the officer uses in the test, defining any parameters used. **[2]**

(b) State the distribution used in the analysis. **[3]**

(c) Carry out the hypothesis test, giving your conclusion in context. **[3]**

13 Records from the 1950s showed that 35% of human babies were born without wisdom teeth. It is believed that as part of the evolutionary process more babies are now born without wisdom teeth. In a random sample of 140 babies, collected in 2020, a researcher found that 61 were born without wisdom teeth.

The researcher made the following statement.

"This shows that the percentage of babies born without wisdom teeth has increased from 35%."

(a) Explain whether this statement can be fully justified. **[1]**

(b) Conduct a hypothesis test at the 5% level to determine whether there is any evidence to suggest that more than 35% of babies are now born without wisdom teeth. **[7]**

14 **Fig. 14.1** shows the curve with equation $y = \dfrac{1}{1+x^2}$, together with 5 rectangles of equal width.

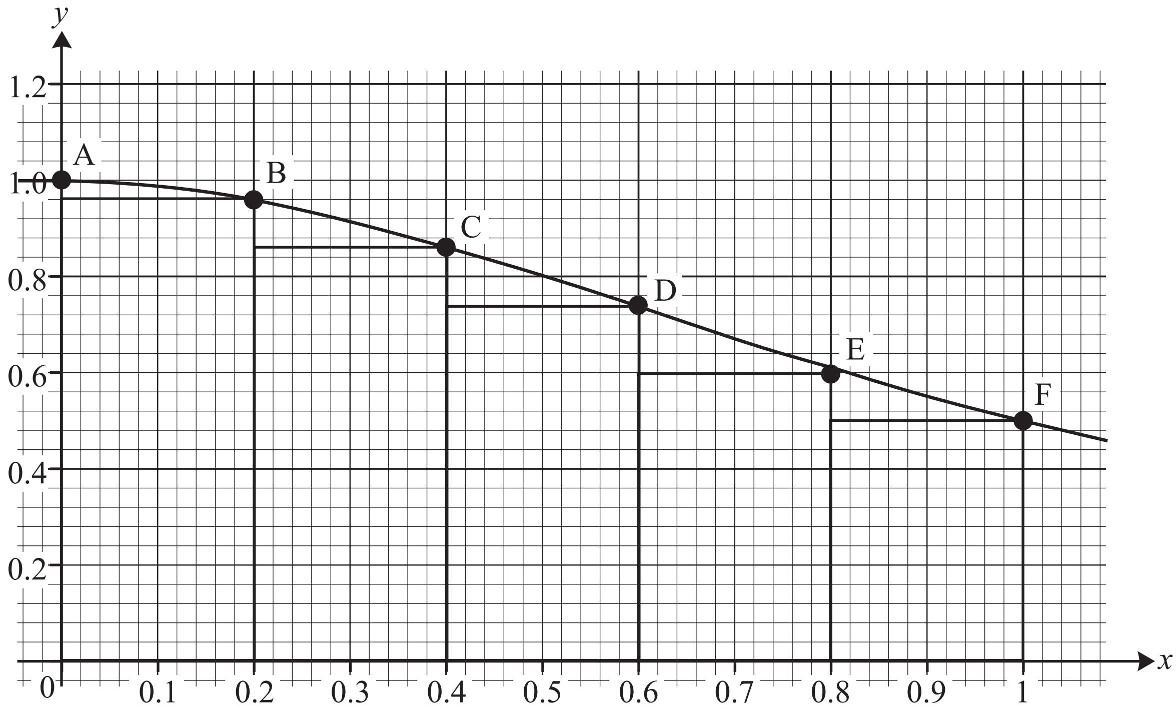


Fig. 14.1

Fig. 14.2 shows the coordinates of the points A, B, C, D, E and F.

Point	A	B	C	D	E	F
x	0	0.2	0.4	0.6	0.8	1
y	1	0.96154	0.86207	0.73529	0.60976	0.5

Fig. 14.2

(a) Use the 5 rectangles shown in **Fig. 14.1** and the information in **Fig. 14.2** to show that a lower bound for $\int_0^1 \frac{1}{1+x^2}\,dx$ is 0.7337, correct to **4** decimal places. **[2]**

(b) Use the 5 rectangles shown in **Fig. 14.1** and the information in **Fig. 14.2** to calculate an upper bound for $\int_0^1 \frac{1}{1+x^2}\,dx$ correct to **4** decimal places. **[2]**

(c) Hence find the length of the interval in which your answers to parts **(a)** and **(b)** indicate the value of $\int_0^1 \frac{1}{1+x^2}\,dx$ lies. **[1]**

Amit uses n rectangles, each of width $\frac{1}{n}$, to calculate upper and lower bounds for $\int_0^1 \frac{1}{1+x^2}\,dx$, using different values of n. His results are shown in **Fig. 14.3**.

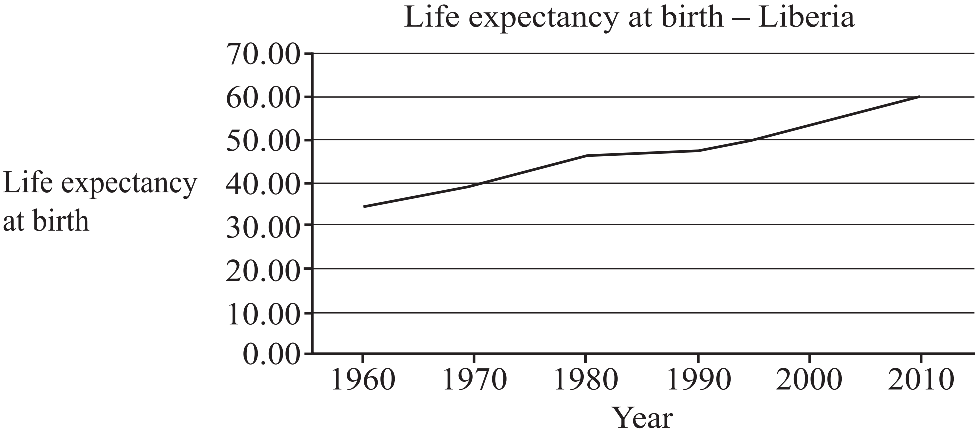
n	10	20	40
upper bound	0.80998	0.79779	0.79162
lower bound	0.75998	0.77279	0.77912

Fig. 14.3

(d) Find the length of the smallest interval in which Amit now knows $\int_0^1 \frac{1}{1+x^2}\,dx$ lies. **[2]**

(e) **Without** doing any calculation, explain how Amit could find a smaller interval which contains the value of $\int_0^1 \frac{1}{1+x^2}\,dx$. **[1]**

15 The pre-release material includes information on life expectancy at birth in countries of the world. **Fig. 15.1** shows the data for Liberia, which is in Africa, together with a time series graph.



1960	1970	1980	1990	2000	2010
34.67	39.25	46.00	47.18	52.42	59.63

Fig. 15.1

Sundip uses the LINEST function on a spreadsheet to model life expectancy as a function of calendar year by a straight line.

The equation of this line is $L = 0.473y - 892$, where L is life expectancy at birth and y is calendar year.

(a) Use this model to find an estimate of the life expectancy at birth in Liberia in 1995. **[1]**

According to the model, the life expectancy at birth in Liberia in 2025 is estimated to be 65.83 years.

(b) Explain whether each of these two estimates is likely to be reliable. **[2]**

(c) Use your knowledge of the pre-release material to explain whether this model could be used to obtain a reliable estimate of the life expectancy at birth in other countries in 1995. **[1]**

Fig. 15.2 shows the life expectancy at birth between 1960 and 2010 for Italy and South Africa.

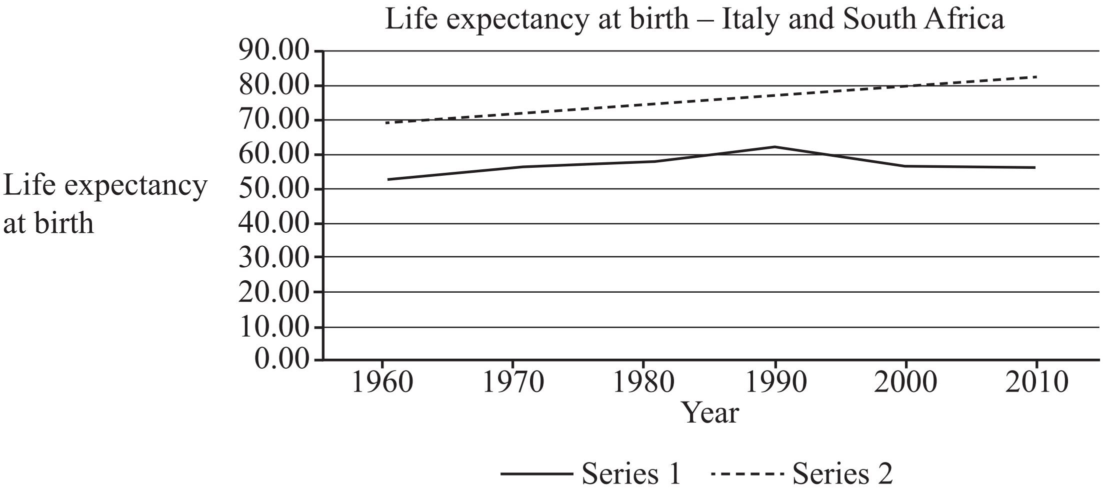


Fig. 15.2

(d) Use your knowledge of the pre-release material to

- Explain whether series 1 or series 2 represents the data for Italy.
- Explain how the data for South Africa differs from the data for most developed countries.

[2]

Turn over for question 15(e)

Sundip is investigating whether there is an association between the wealth of a country and life expectancy at birth in that country. As part of her analysis she draws a scatter diagram of GDP per capita in US $ and life expectancy at birth in 2010 for all the countries in Europe for which data is available. She accidentally includes the data for the Central African Republic. The diagram is shown in **Fig. 15.3**.

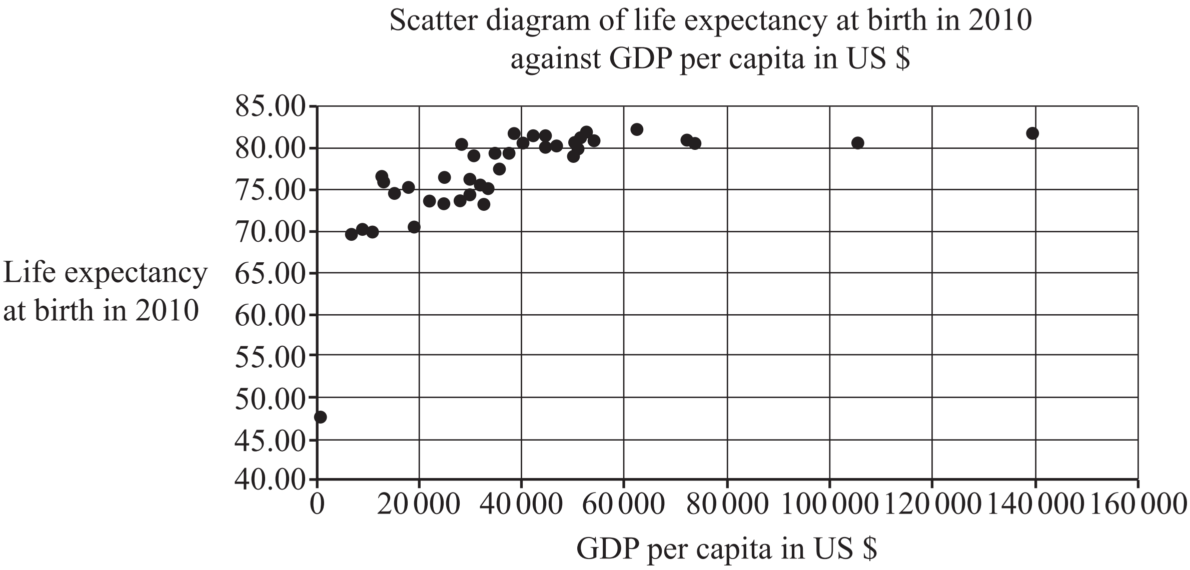


Fig. 15.3

(e) On the copy of **Fig. 15.3** in the Printed Answer Booklet, use your knowledge of the pre-release material to circle the point representing the data for the Central African Republic. **[1]**

Sundip states that as GDP per capita increases, life expectancy at birth increases.

(f) Explain to what extent the information in **Fig. 15.3** supports Sundip's statement. **[2]**

16 The equation of a curve is

$$y = 6x^4 + 8x^3 - 21x^2 + 12x - 6.$$

(a) **In this question you must show detailed reasoning.**

Determine

- The coordinates of the stationary points on the curve.
- The nature of the stationary points on the curve.
- The x-coordinate of the non-stationary point of inflection on the curve. **[12]**

(b) On the axes in the Printed Answer Booklet, sketch the curve whose equation is

$$y = 6x^4 + 8x^3 - 21x^2 + 12x - 6.$$ **[3]**

END OF QUESTION PAPER

BLANK PAGE

