Oxford Cambridge and RSA

GCE

Further Mathematics B (MEI)

Y420/01: Core Pure

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Further Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

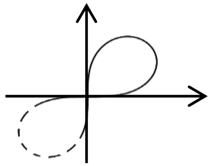
h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs		Guidance
1			$\sum_{r=1}^{n}(2r^2-1)=\frac{1}{3}n(n+1)(2n+1)-n$	B1	2.5	$\frac{1}{3}n(n+1)(2n+1)$...	
				B1	1.1b	… $-n$	
			$=\frac{1}{3}n(2n^2+3n-2)$	B1	1.1b	factoring out n	correctly
			$=\frac{1}{3}n(2n-1)(n+2)$	B1cao [4]	1.1b	allow $\frac{2}{6}n(2n-1)(n+2)$, etc	
2			$(\mathbf{i}+2\mathbf{j}+c\mathbf{k}).(2\mathbf{i}-c\mathbf{j}+6\mathbf{k})=0$ $\Rightarrow 2-2c+6c=0$ $\Rightarrow c=-½$	M1 A1 A1 [3]	1.1a 1.1b 1.1b	scalar product = 0	
3	(a)		$\mathbf{AB}=\begin{pmatrix}3&1\\2&1\end{pmatrix}\begin{pmatrix}k&1\\2&0\end{pmatrix}=\begin{pmatrix}3k+2&3\\2k+2&2\end{pmatrix}$	B1	1.1b		
			$(\mathbf{AB})^{-1}=-\frac{1}{2}\begin{pmatrix}2&-3\\-2k-2&3k+2\end{pmatrix}$	B1ft	1.1b	ft their **AB** provided det ≠ 0	[isw]
			$\mathbf{A}^{-1}=\begin{pmatrix}1&-1\\-2&3\end{pmatrix}$	B1	1.1b		
			$\mathbf{B}^{-1}=-\frac{1}{2}\begin{pmatrix}0&-1\\-2&k\end{pmatrix}$	B1	1.1b		
			$\mathbf{B}^{-1}\mathbf{A}^{-1}=-\frac{1}{2}\begin{pmatrix}2&-3\\-2k-2&3k+2\end{pmatrix}$ [so $(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$]	B1 [5]	2.2a		

Question			Answer	Marks	AOs		Guidance
3	(b)		**BA** $=\begin{pmatrix} 3k+2 & k+1 \\ 6 & 2 \end{pmatrix}$	B1	1.1b		
			AB = **BA** when $k = 2$ [and not otherwise]	B1 [2]	2.3		
4			**DR**				
			$V=\int_0^{\frac{\pi}{2}} \pi \sec^2 \frac{1}{2}x \,\mathrm{d}x$	B1	1.1b	correct integral and limits	
			$=\pi\left[2\tan\frac{1}{2}x\right]_0^{\frac{\pi}{2}}$	B1	1.1b	$2\tan\frac{1}{2}x$	condone π missing
			$= \pi(2\tan \frac{1}{4}\pi - 0) = 2\pi$	B1cao [3]	1.1b	unsupported B0	
5			$\cos 2x = 1-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}-\frac{(2x)^6}{6!}+\ldots$	M1	1.1a	at least 3 terms correct	or good attempt from 1st principles
			$=1-2x^2+\frac{2}{3}x^4-\frac{4}{45}x^6+\ldots$	A1	1.1b	Allow unsimplified fractions (without factorials)	
			$\sin^2 x = \frac{1}{2}(1-\cos 2x)$	M1	3.1a		
			$=\frac{1}{2}(1-1+2x^2-\frac{2}{3}x^4+\frac{4}{45}x^6+\ldots)$				
			$=x^2-\frac{1}{3}x^4+\frac{2}{45}x^6+\ldots$ so $a=1,\ b=-\frac{1}{3},\ c=\frac{2}{45}$	A2,1,0 [5]	1.1b		

Question			Answer	Marks	AOs		Guidance
6			**DR** $I=\int_2^\infty \frac{1}{4+x^2}\,\mathrm{d}x=\left[\frac{1}{2}\arctan\frac{x}{2}\right]_2^\infty$ as $x\to\infty$, arctan ½ $x\to$ ½ π $I=\frac{1}{8}\pi$	**B1** **B2** **B1** **[4]**	**1.1b** **2.4,2.2a** **1.1b**	$\left[\frac{1}{2}\arctan\frac{x}{2}\right]$ if ½ π only B1 0.393 or better	 (soi) allow B1 if unsupported
7	**(a)**		$(x^2+y^2)^2=2c^2xy \Rightarrow (r^2)^2=2c^2r\cos\theta r\sin\theta$ $\Rightarrow r^2=2c^2\cos\theta\sin\theta=c^2\sin 2\theta$ *	**M1** **A1** **[2]**	**1.1b** **2.2a**	substituting for r^2, x and y **NB AG**	
7	**(b)**			**B1** **B1*** **B1dep** **[3]**	**1.1b** **1.1b** **2.5**	one loop shown both shown (no extras) –ve r with broken line	allow sep diags in correct quadrant dep B1*
7	**(c)**		$A=\int_0^{\frac{\pi}{2}}\frac{1}{2}c^2\sin 2\theta\,\mathrm{d}\theta$ $=\left[-\frac{1}{4}c^2\cos 2\theta\right]_0^{\frac{\pi}{2}}$ $=\frac{1}{2}c^2$	**B1** **B1** **B1cao** **[3]**	**1.1a** **1.1b** **1.1b**	condone missing dθ $\int\sin 2\theta\,\mathrm{d}\theta=-\frac{1}{2}\cos 2\theta$	limits soi

Question			Answer	Marks	AOs		Guidance
8	(a)		**DR**				
			$\alpha.\frac{1}{\alpha}.\beta = 2 \Rightarrow \beta = 2$	**M1** **A1**	**3.1a** **1.1b**	product of roots used $\beta = 2$	
			$\alpha + \frac{1}{\alpha} + \beta = 1$	**M1**	**1.1b**	sum of roots used	or $(x-2)(x^2+x+1) = 0$ M1A1
			$\Rightarrow \alpha^2 + \alpha + 1 = 0$	**A1**	**1.1b**	or equivalent quadratic (with $\beta = 2$)	
			$\Rightarrow \alpha = \frac{-1 \pm \sqrt{-3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}$	**M1**	**1.1b**	solving their quadratic	$x = \frac{-1 \pm \sqrt{-3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}$
			roots are [2], $-\frac{1}{2}+\frac{\sqrt{3}}{2}\mathrm{i}, -\frac{1}{2}-\frac{\sqrt{3}}{2}\mathrm{i}$	**A1**	**1.1b**		
				[6]			
8	(b)		$k = \alpha.\frac{1}{\alpha} + \alpha\beta + \frac{1}{\alpha}\beta$	**M1**	**1.1a**	k = product of root pairs	or $(x-2)(x^2+x+1) \Rightarrow k = -1$
			$= 1 + 2(\alpha + \frac{1}{\alpha}) = 1 - 2 = -1$	**A1** **[2]**	**1.1b**	or by direct substitution	or by factor theorem
9			When $n = 1$, $5^1 + 2\times11^1 = 27$ div by 3 Assume $u_k = 5^k + 2\times11^k$ is div by 3 $u_{k+1} = 5^{k+1} + 2\times11^{k+1}$	**B1*** **M1** **M1**	**2.1** **2.1**	 or $5^k + 2\times11^k = 3m$	
			$= 5(u_k - 2\times11^k) + 22\times11^k$ $= 5u_k + 12\times11^k$	**M1** **A1**	 **1.1b**	substituting for 5^k or $15m + 12\times11^k$	
			or $u_{k+1} = 5^{k+1} + 11(u_k - 5^k)$ $= 11u_k - 6\times5^k$	**M1** **A1**		substituting for 11^k or $33m - 6\times5^k$	$5.5^k + 11(3m - 5^k)$
			or $u_{k+1} + u_k = 5^{k+1} + 2\times11^{k+1} + 5^k + 2\times11^k$ $= 6 \times 5^k + 24 \times 11^k$	**M1** **A1**	**1.1b**	adding u_k to u_{k+1}	
			As u_k div by 3, u_{k+1} div by 3 So if true for $n = k$, true for $n = k+1$. As true for $n = 1$, true for all positive integers n	**A1*** **A1dep** **[7]**	**2.2a** **2.4**	 dep * marks	

Question			Answer	Marks	AOs		Guidance
10	(a)		**DR** $(-1 + i)^3 = (-1)^3 + 3(-1)^2 i + 3(-1)i^2 + i^3$ $= 2 + 2i$ [so $a = 2$ and $b = 2$]	 **M1** **A1** **A1**	 **3.1a** **1.1b** **2.2a**	 expanding $(-1 + i)^3$ correct expression	 or $-2i(-1 + i)$
			Alternative solution $-1 + i = \sqrt{2}(\cos 3\pi/4 + i \sin 3\pi/4)$ $(-1 + i)^3 = 2\sqrt{2}(\cos 9\pi/4 + i \sin 9\pi/4)$ $= 2\sqrt{2}(1/\sqrt{2} + 1/\sqrt{2}) = 2 + 2i$	 **B1** **M1** **A1** **[3]**		 $\lvert z\rvert = \sqrt{2}$, $\arg z = 3\pi/4$ $\lvert z^3\rvert = \lvert z\rvert^3$, $\arg(z^3) = 3\arg z$	 or $\sqrt{2}\, e^{3i\pi/4}$
10	(b)		 $z = \sqrt{2}e^{i\pi/12}, \sqrt{2}e^{3i\pi/4}, \sqrt{2}e^{-7i\pi/12}$	**B1** **B1B1B1** **[4]**	**2.5** **1.1b**	3 roots with modulus $\sqrt{2}$ oe oe, e.g. $8^{1/6}e^{17i\pi/12}$, etc	oe eg $8^{1/6}$ $0 < \arg < 2\pi$ or $-\pi < \arg < \pi$
10	(c)		The statement only applies to polynomial equations with real coefficients.	 **B1** **[1]**	 **2.3**		

Question			Answer	Marks	AOs		Guidance
11	(a)		**M₁** rotation through $\cos^{-1}(3/5)$ or 53.1° or 0.927 rads anti-clockwise about O **M₂** reflection in x-axis	**M1** **A1** **A1** **B1** **[4]**	**3.1a** **1.1b** **1.2** **1.2**	 oe e.g. $\sin^{-1}(4/5)$, $\tan^{-1}(4/3)$ or positive rotation about O or Ox or $y = 0$	 53° or 0.93 rads or better
11	(b)		$\mathbf{M_3} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}$	**B1**	**1.1b**		
			$\begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow \frac{3}{5}x + \frac{4}{5}y = x,\ \frac{4}{5}x - \frac{3}{5}y = y$ $\Rightarrow y = \frac{1}{2}x$ so $y = \frac{1}{2}x$ is mirror line	**M1** **A2** **A1**	**3.1a** **1.1b** **2.2a**	attempt to find invariant points either or both accept valid geometric args	or inv line $y = mx$ [$+c$] $2m^2+3m-2=0$ A1 $\Rightarrow m = \frac{1}{2}, -2$ A1
			Alternative solution $\frac{1}{1+m^2}\begin{pmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}$ $\Rightarrow 2m^2 - 5m + 2 = 0$ $\Rightarrow m = \frac{1}{2}$	 **M1** **A1** **A2** **[5]**		 must discount $m = 2$	
11	(c)		$\mathbf{M_4} = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & -\frac{3}{5} \end{pmatrix}$	**B1**	**1.1b**		
			$\mathbf{M_4} \neq \mathbf{M_3}$ [so can't represent same reflection] so mirror line cannot be the same, and statement is incorrect	**M1** **A1**	**3.1a** **2.4**	or attempt to find mirror line as in part **(b)** $\Rightarrow y = -\frac{1}{2}x$, so statement is incorrect	**M₄** must be different

Question			Answer	Marks	AOs		Guidance
12			L_1 and L_3: $\lambda = -4 + 5\nu$, $-8 + 10\nu = 1 + \nu$ $\Rightarrow \nu = 1$, $\lambda = 1$, so meet at (2, 3, 1) L_2 and L_3 meet at (1, 2, −4) L_1 and L_2 meet at origin	M1 B1 B1 B1 (4)	1.1b 2.2a 2.2a 2.2a	attempt to solve (any pair) by solving or inspection by solving or inspection by solving or inspection	 soi
			$\cos\theta = \dfrac{(2\mathbf{i}+3\mathbf{j}+\mathbf{k}).(\mathbf{i}+2\mathbf{j}-4\mathbf{k})}{\sqrt{2^2+3^2+1^2}\sqrt{1^2+2^2+(-4)^2}}$ $= \dfrac{4}{\sqrt{14}\sqrt{21}}$ $\theta = 76.5°$ (1.335)	M1 A1 A1 (3)	3.1a 1.1b 1.1b	 or $\cos^{-1}\dfrac{10}{\sqrt{14}\sqrt{27}}, \cos^{-1}\dfrac{17}{\sqrt{21}\sqrt{27}}$ 59.0° (1.030), 44.4°(0.775)	or cosine rule sides √21, √14, √27 any correct angle
			Area = ½ √14 √21 sin 76.5° = 8.34 [units^2]	M1 A1 (2)	1.1a 3.2a	or ½ √14 √27 sin 59.0° art 8.3 or √278/2	or ½ √21 √27 sin 44.4°
			Alternative solution $(2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(\mathbf{i}+2\mathbf{j}-4\mathbf{k})$ $=-14\mathbf{i}+9\mathbf{j}+\mathbf{k}$	 M1 A1 (2)		using cross product $(-2\mathbf{i}-3\mathbf{j}-\mathbf{k})\times(-\mathbf{i}-\mathbf{j}-5\mathbf{k})$ $=14\mathbf{i}-9\mathbf{j}-\mathbf{k}$	 $(-\mathbf{i}-2\mathbf{j}+4\mathbf{k})\times(\mathbf{i}+\mathbf{j}+5\mathbf{k})$ $=-14\mathbf{i}+9\mathbf{j}+\mathbf{k}$
			Area = ½ × √(14^2 + 9^2 + 1^2) = ½ √278 = 8.34 [units^2]	M1 A2 (3)		or ½ base × height art 8.34	½ ×√14 ×√287/√14, etc
				[9]			

Question			Answer	Marks	AOs		Guidance
13	(a)		$y = \text{arcosh}\, x = \ln(x + \sqrt{x^2 - 1})$	M1		$\frac{d}{du}(\ln u) = \frac{1}{u}$	or $e^y = x + \sqrt{(x^2 - 1)}$
				M1	1.1b	chain rule on $\sqrt{(x^2 - 1)}$	$e^y\, dy/dx = \ldots$ **M1** $= 1 + x(x^2-1)^{-1/2}$ **M1** [substituting for e^y]
			$\frac{dy}{dx} = \frac{1 + x(x^2-1)^{-1/2}}{(x + (x^2-1)^{1/2})}$	A1	1.1b	correct expression	
			$= \frac{(x^2-1)^{1/2} + x}{(x + (x^2-1)^{1/2})(x^2-1)^{1/2}}$	A1	2.1	or $\frac{[(x^2-1)^{1/2} + x](x^2-1)^{-1/2}}{(x + (x^2-1)^{1/2})}$	
			$= \frac{1}{(x^2-1)^{1/2}}$ *	A1cao [5]	2.2a	**NB AG**	
13	(b)		let u = arcosh x, $u' = 1/\sqrt{(x^2 - 1)}$, $v' = 1$, $v = x$	M1	3.1a	integration by parts	
			$\int_1^2 \text{arcosh}\, x\, dx = [x\, \text{arcosh}\, x]_1^2 - \int_1^2 \frac{x}{\sqrt{x^2-1}} dx$	A1	1.1b	ignore limits	
				M1	1.1b	substitution or inspection	
			$= \left[x\, \text{arcosh}\, x - \sqrt{x^2-1} \right]$	A1	1.1b	$\int \frac{x}{\sqrt{x^2-1}} dx = \sqrt{x^2-1}$	
			= 2arcosh 2 – arcosh 1 – $\sqrt{3}$ = 2ln(2 + $\sqrt{3}$) – $\sqrt{3}$	A1cao		oe e.g. ln(7 + 4$\sqrt{3}$) – $\sqrt{3}$	isw, not ln 1
			Alternative solution Let x = cosh u, dx = sinh u du				
			$\int \text{arcosh}\, x\, dx = \int u \sinh u\, du$	M1			
			$= [u \cosh u] - \int \cosh u\, du$	M1A1		integration by parts	
			$= [u \cosh u - \sinh u]_{ar\cosh 1}^{ar\cosh 2}$	A1		limits not needed	
			= 2ln(2 +$\sqrt{3}$) – $\sqrt{3}$	A1cao [5]		oe e.g. ln(7 + 4$\sqrt{3}$) – $\sqrt{3}$	[isw] , not ln 1
13	(c)		arcosh x does not exist for $x < 1$	B1 [1]	2.4	or $\sqrt{(x^2 - 1)} = \sqrt{(-1)}$ not real, so ln(x+$\sqrt{(x^2-1)}$) is not real	accept other valid arguments

Question			Answer	Marks	AOs		Guidance
14	(a)	(i)	let $\mathbf{M} = \begin{pmatrix} -1 & a & 0 \\ 2 & 3 & 1 \\ 1 & b & 1 \end{pmatrix}$	**M1**	**3.1a**	finding matrix of coefficients	
			det $\mathbf{M} = b - 3 - a$	**B1**	**1.1b**		
			det $\mathbf{M} = 0$	**M1**	**1.1b**		
			$\Rightarrow b = a + 3$	**A1** **[4]**	**3.2a**		
14	(a)	(ii)	$x = ay - 2 \Rightarrow (2a+3)y + z = 1$ $(a + b)y + z = c + 2$, $b = a+3$	**M1**	**3.1a**	reduce system to 2 equations in 2 variables	one including c
			$\Rightarrow (2a+3)y + z = c+2$	**M1**	**3.1a**	use $b = a + 3$ to find value of c for consistency	
			$\Rightarrow c = -1$	**A1cao** **[3]**	**3.2a**		or other valid method
14	(b)		$\mathbf{M}^{-1} = -\frac{1}{3}\begin{pmatrix} 3-a & -a & a \\ -1 & -1 & 1 \\ 2a-3 & 2a & -3-2a \end{pmatrix}$	**M1** **A2** **M1**	**3.1a** **1.1b** **1.1b**	attempt to find $\mathbf{M}^{-1}$ **A1** any 6 entries correct $\times$ 1/their det	
			$\mathbf{M}^{-1}\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} 6+2a \\ 2 \\ -4a-9 \end{pmatrix}$	**M1**	**1.1b**	pre-multiplying by their $\mathbf{M}^{-1}$	
			coordinates are $(-\frac{6+2a}{3}, -\frac{2}{3}, \frac{4a+9}{3})$	**A1cao** **[6]**	**3.2a**	accept in vector form	

Question			Answer	Marks	AOs		Guidance
			Alternative solution				
			$-x + ay = 2,\ x + ay + z = 1$				
			$\Rightarrow 2x + z = -1,\ z = -2x - 1$	**M1**		eliminate one variable	from 2 equations
			$-x + ay = 2 \Rightarrow y = (2+x)/a$	**M1**		eliminate another variable	
			$\Rightarrow\ 2x + \dfrac{3x+6}{a} - 1 - 2x = -3$	**M1**		substitute into 3rd eqn	to get eqn in one unknown
			$\Rightarrow x = -\dfrac{2a+6}{3}\quad y = -\dfrac{2}{3}\ ,\ z = = \dfrac{4a+9}{3}$	**A3** **[6]**			
15			**DR**				
			$\displaystyle\int_{\frac{3}{4}}^{\frac{3}{2}} \frac{1}{\sqrt{4x^2-4x+2}}\,\mathrm{d}x = \int_{\frac{3}{4}}^{\frac{3}{2}} \frac{1}{\sqrt{(2x-1)^2+1}}\,\mathrm{d}x$	**M1**	**3.1a**	attempt to complete the square	or $\displaystyle\frac{1}{2}\int_{\frac{3}{4}}^{\frac{3}{2}} \frac{1}{\sqrt{(x-1/2)^2+1/4}}\,\mathrm{d}x$
			$\displaystyle= \left[\frac{1}{2}\mathrm{arsinh}(2x-1)\right]_{\frac{3}{4}}^{\frac{3}{2}}$	**M1** **A1** **(3)**	**1.1b** **1.1b**	arsinh($2x$ – 1) (oe) × ½ oe e.g. ln form	$\displaystyle= \left[\frac{1}{2}\mathrm{ar}\sinh u\right]_{\frac{1}{2}}^{2}$ if $u = 2x-1$
			$\displaystyle= \frac{1}{2}[\mathrm{arsinh}(2) - \mathrm{arsinh}(\frac{1}{2})]$				
			$\displaystyle= \frac{1}{2}[\ln(2+\sqrt{5}) - \ln(\frac{1}{2}+\frac{\sqrt{5}}{2})]$	**M1** **A1**	**1.1b**	arsinh x=ln(x+√(x^2 + 1)) correct expression	(used)
			$\displaystyle= \frac{1}{2}\ln\frac{2(\sqrt{5}+2)}{\sqrt{5}+1}$	**M1**	**2.1**	combining lns	
			$\displaystyle= \frac{1}{2}\ln\frac{2(\sqrt{5}+2)(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)}$	**M1**	**2.1**	rationalizing denominator (must be seen)	
			$\displaystyle= \frac{1}{2}\ln\frac{(\sqrt{5}+3)}{2}\ *$	**A1cao** **(5)**	**2.1**	**NB AG**	
				[8]			

Question			Answer	Marks	AOs		Guidance
16	(a)		$(2-e^{i\theta})(2-e^{-i\theta}) = 4 - 2(e^{i\theta}+e^{-i\theta}) + 1$	M1	1.1b	$e^{i\theta}.e^{-i\theta} = 1$	
			$= 5 - 2.2\cos\theta$	M1	1.1b	$e^{i\theta}+e^{-i\theta} = 2\cos\theta$ used	
			$= 5 - 4\cos\theta$*	A1	2.2a	**NB AG**	
				[3]			
16	(b)		$C+iS = \frac{1}{2}e^{i\theta}+\frac{1}{4}e^{2i\theta}+\frac{1}{8}e^{3i\theta}+\ldots+\frac{1}{2^n}e^{ni\theta}$	M1	3.1a	at least 2 terms	
				A1	2.1	correct (nth term soi)	
			$=\dfrac{\frac{1}{2}e^{i\theta}\left(1-(\frac{1}{2}e^{i\theta})^n\right)}{1-\frac{1}{2}e^{i\theta}}$	M1	2.1	sum of GP (condone S_∞)	
				A1	1.1b	correct expression	
				(4)			
			$=\dfrac{e^{i\theta}\left(1-(\frac{1}{2}e^{i\theta})^n\right)(2-e^{-i\theta})}{(2-e^{i\theta})(2-e^{-i\theta})}$	M1	2.1	× top and bottom by complex conjugate	
			$=\dfrac{e^{i\theta}(2-\frac{1}{2^{n-1}}e^{ni\theta}-e^{-i\theta}+\frac{1}{2^n}e^{(n-1)i\theta})}{5-4\cos\theta}$	M1	2.1	expand brackets	allow 1 slip, not on S_∞
				A1			
				(3)			
			$=\dfrac{2^{n+1}e^{i\theta}-2e^{(n+1)i\theta}-2^n+e^{ni\theta}}{2^n(5-4\cos\theta)}$				
			$C = \mathrm{Re}\left(\dfrac{2^{n+1}e^{i\theta}-2e^{(n+1)i\theta}-2^n+e^{ni\theta}}{2^n(5-4\cos\theta)}\right)$	M1	2.1	taking real part	not on S_∞
			$=\dfrac{2^n(2\cos\theta-1)-2\cos(n+1)\theta+\cos n\theta}{2^n(5-4\cos\theta)}$ *	A1cao	2.2a	**NB AG**	need evidence of clearing subsidiary denominators
				(2)			
				[9]			

Question			Answer	Marks	AOs		Guidance
			Alternative solution				
			$C=\frac{1}{2}(\frac{e^{i\theta}+e^{-i\theta}}{2})e^{i\theta}+\frac{1}{4}(\frac{e^{2i\theta}+e^{-2i\theta}}{2})+...+\frac{1}{2^n}($	**M1**		subsitituting for cosines in terms of $e^{i\theta}$s	
			$=\frac{\frac{1}{2}e^{i\theta}\left(1-(\frac{1}{2}e^{i\theta})^n\right)}{1-\frac{1}{2}e^{i\theta}}+\frac{\frac{1}{2}e^{-i\theta}\left(1-(\frac{1}{2}e^{-i\theta})^n\right)}{1-\frac{1}{2}e^{-i\theta}}$	**M1** **A1**		sum of GP	
			$=\frac{e^{i\theta}\left(1-(\frac{1}{2}e^{i\theta})^n\right)(2-e^{-i\theta})+e^{-i\theta}\left(1-(\frac{1}{2}e^{-i\theta})^n\right)(2-e^{i\theta})}{(2-e^{i\theta})(2-e^{-i\theta})}$	**M1** **A1**		combining fractions	
				M1 **A1**		expanding	
			$=\frac{e^{i\theta}+e^{-i\theta}-\frac{1}{2^n}(e^{(n+1)i\theta}+e^{-(n+1)i\theta})-2+\frac{1}{2^{n+1}}(e^{ni\theta}+e^{-ni\theta})}{(2-e^{i\theta})(2-e^{-i\theta})}$	**M1**		expressing in cosines	
			$=\frac{2\cos\theta-\frac{1}{2^{n-1}}\cos(n+1)\theta-2+\frac{1}{2^n}\cos n\theta}{5-4\cos\theta}$	**A1** **[9]**		**NB AG**	
			$=\frac{2^n(2\cos\theta-1)-2\cos(n+1)\theta+\cos n\theta}{2^n(5-4\cos\theta)}$ *				

Question			Answer	Marks	AOs		Guidance
17	(a)		The resistance force is likely to increase with velocity.	**B1** **[1]**	**3.5b**	allow 'proportional to', 'varies with'	
17	(b)		$m\frac{\mathrm{d}v}{\mathrm{d}t}=-2m-0.1mv \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}t}+0.1v=-2$	**B1** **[1]**	**3.3**	[by Newton's 2nd Law]	
17	(c)	(i)	IF $\mathrm{e}^{0.1t}$ $\Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\left(v\mathrm{e}^{0.1t}\right)=-2\mathrm{e}^{0.1t}$ $\Rightarrow v\mathrm{e}^{0.1t}=\int -2\mathrm{e}^{0.1t}\,\mathrm{d}t=-20\mathrm{e}^{0.1t}+c$ when $t = 10$, $v = 0 \Rightarrow c = 20\mathrm{e}$ $\Rightarrow v = 20(\mathrm{e}^{1-0.1t} - 1)$	**M1** **M1** **A1** **M1** **A1cao**	**1.1a** **1.1b** **1.1b** **3.1b** **3.4**	must be correct substituting t=10, v = 0 or $54.4\mathrm{e}^{-0.1t} - 20$	
			Alternative solution $\int\frac{\mathrm{d}v}{2+0.1v}=-\int \mathrm{d}t$ $\Rightarrow 10\ln(2+0.1v)=-t+c$ When $t = 10$, $v = 0 \Rightarrow c = 10\ln 2 + 10$ $\Rightarrow \ln(2+0.1v) = \ln 2 + 1 - 0.1t$ $2 + 0.1v = \mathrm{e}^{\ln 2 + 1 - 0.1t} = 2\mathrm{e}^{1 - 0.1t}$ $\Rightarrow v = 20(\mathrm{e}^{1 - 0.1t} - 1)$	**M1** **A1** **M1** **M1** **A1cao**		substituting t=10, v = 0 anti-logging	
			Alternative solution AE $\lambda + 0.1 = 0 \Rightarrow$ cf $v = A\mathrm{e}^{-0.1t}$ PI $v = k \Rightarrow k = -20$ GS $v = A\mathrm{e}^{-0.1t} - 20$ When $t = 0$. $v = 10$: $0 = A\mathrm{e}^{-1} - 20 \Rightarrow A = 20\mathrm{e}$ $\Rightarrow v = 20(\mathrm{e}^{1-0.1t} - 1)$	**M1** **B1** **A1** **M1** **A1cao**		substituting t=10, v = 0	
				[5]			

Question			Answer	Marks	AOs		Guidance
17	(c)	(ii)	When $t = 5$, $v = 20(e^{0.5} - 1) = 12.97$ m s^{-1}.	**B1** **[1]**	**3.5a**	13 or better	
17	(d)		$m\dfrac{dv}{dt} = ct - 0.1mv \Rightarrow \dfrac{dv}{dt} + 0.1v = \lambda t$ where $\lambda = \dfrac{c}{m}$	**B1** **[1]**	**3.3**	[by Newton's 2nd Law]	
17	(e)	(i)	$\dfrac{d}{dt}\left(v e^{0.1t}\right) = \lambda t e^{0.1t}$	**M1**	**2.1**		
			$\Rightarrow v e^{0.1t} = \int \lambda t e^{0.1t}\, dt = 10\lambda t e^{0.1t} - \int 10\lambda e^{0.1t}\, dt$	**M1**	**2.1**	integrating by parts	
			$\Rightarrow v e^{0.1t} = 10\lambda t e^{0.1t} - 100\lambda e^{0.1t} + c$	**A1**	**2.1**		
			When $t = 0$, $v = 0 \Rightarrow c = 100\lambda$	**M1**	**3.1b**	substituting $t = 0$, $v = 0$	
			$\Rightarrow v = 10\lambda(t - 10 + 10e^{-0.1t})$ *	**A1**	**2.1**	**NB AG**	
			Alternative solution				
			CF $v = Ae^{-0.1t}$	**M1**			
			PI $v = Ct + D$	**M1**			
			$C + 0.1(Ct + D) = \lambda t \Rightarrow C = 10\lambda, D = -100\lambda$	**A1**			
			GS $v = Ae^{-0.1t} + 10\lambda t - 100\lambda$				
			$0 = A - 100\lambda \Rightarrow A = 100\lambda$	**M1**		substituting $t = 0$, $v = 0$	
			$v = 10\lambda(t - 10 + 10e^{-0.1t})$ *	**A1cao** **[5]**		**NB AG**	
17	(e)	(ii)	When $t = 5$, $20(e^{0.5} - 1) = 10\lambda(10e^{-0.5} - 5)$	**M1**	**3.1b**	subst $t = 5$ and equating to their v when $t = 5$	
			$\Rightarrow \lambda = 1.218$	**A1** **[2]**	**1.1b**	1.2 or better	

Question			Answer	Marks	AOs		Guidance
17	(f)		$s_1 = \int_0^5 10\lambda(t - 10 + 10\mathrm{e}^{-0.1t})\,\mathrm{d}t$	M1	3.1b	integrating v between 0, 5	
			$= 10\lambda\left[\frac{1}{2}t^2 - 10t - 100\mathrm{e}^{-0.1t}\right]_0^5$	B1	1.1b	$\left[\frac{1}{2}t^2 - 10t - 100\mathrm{e}^{-0.1t}\right]$	
			$= 12.18(12.5 + 50 - 100e^{-0.5}) = 22.49\text{(m)}$	A1	1.1b	art 22.5 (soi)	
			$s_2 = \int_5^{10} 20(\mathrm{e}^{1-0.1t} - 1)\,\mathrm{d}t$	M1	3.1b	integrating their v between 5 and 10	
			$= 20\left[-10\mathrm{e}^{1-0.1t} - t\right]_5^{10}$				
			$= 20(-15 + 10e^{0.5}) = 29.74\text{(m)}$	A1	1.1b	art 29.7 (soi)	
			Total distance = 52 m	A1cao [6]	3.2b		

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