Oxford Cambridge and RSA

Monday 13 May 2019 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions

1 **In this question you must show detailed reasoning.**

Find $\sum_{r=1}^{100}\left(\frac{1}{r}-\frac{1}{r+2}\right)$, giving your answer correct to 4 decimal places. **[3]**

2 The roots of the equation $3x^2-x+2=0$ are α and β.
Find a quadratic equation with integer coefficients whose roots are $2\alpha-3$ and $2\beta-3$. **[3]**

3 **In this question you must show detailed reasoning.**

A and **B** are matrices such that $\mathbf{B}^{-1}\mathbf{A}^{-1}=\begin{pmatrix}2 & 1\\ -1 & 1\end{pmatrix}$.

(a) Find **AB**. **[3]**

(b) Given that $\mathbf{A}=\begin{pmatrix}\frac{1}{3} & 1\\ 0 & 1\end{pmatrix}$, find **B**. **[3]**

4 **(a)** Find $\mathbf{M}^{-1}$, where $\mathbf{M}=\begin{pmatrix}1 & 2 & 3\\ -1 & 1 & 2\\ -2 & 1 & 2\end{pmatrix}$. **[1]**

(b) Hence find, in terms of the constant k, the point of intersection of the planes

$$\begin{aligned} x+2y+3z&=19,\\ -x+\ \ y+2z&=4,\\ -2x+\ \ y+2z&=k. \end{aligned}$$
[3]

(c) **In this question you must show detailed reasoning.**

Find the acute angle between the planes $x+2y+3z=19$ and $-x+y+2z=4$. **[4]**

5 Prove by induction that, for all positive integers n, $\sum_{r=1}^{n}\frac{1}{3^r}=\frac{1}{2}\left(1-\frac{1}{3^n}\right)$. **[6]**

6 A linear transformation T of the x-y plane has an associated matrix $\mathbf{M}$, where $\mathbf{M} = \begin{pmatrix} \lambda & k \\ 1 & \lambda - k \end{pmatrix}$, and λ and k are real constants.

(a) You are given that $\det\mathbf{M} > 0$ for all values of λ.

(i) Find the range of possible values of k. **[3]**

(ii) What is the significance of the condition $\det\mathbf{M} > 0$ for the transformation T? **[1]**

For the remainder of this question, take $k = -2$.

(b) Determine whether there are any lines through the origin that are invariant lines for the transformation T. **[4]**

(c) The transformation T is applied to a triangle with area 3 units^2. The area of the resulting image triangle is 15 units^2.
Find the possible values of λ. **[3]**

7 **(a)** Sketch on a single Argand diagram

(i) the set of points for which $|z - 1 - 3\text{i}| = 3$, **[3]**

(ii) the set of points for which $\arg(z+4) = \frac{1}{4}\pi$. **[3]**

(b) Find, in exact form, the two values of z for which $|z - 1 - 3\text{i}| = 3$ and $\arg(z+4) = \frac{1}{4}\pi$. **[6]**

8 **In this question you must show detailed reasoning.**

You are given that i is a root of the equation $z^4 - 2z^3 + 3z^2 + az + b = 0$, where a and b are real constants.

(a) Show that $a = -2$ and $b = 2$. **[4]**

(b) Find the other roots of this equation. **[7]**

END OF QUESTION PAPER