OCR
Oxford Cambridge and RSA

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure
Question Paper

Monday 14 May 2018 – Afternoon
Time allowed: 1 hour 15 minutes

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You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The matrices **A**, **B** and **C** are defined as follows:

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 0 & 3 \\ 1 & -1 & 3 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & 3 \end{pmatrix}.$$

Calculate all possible products formed from two of these three matrices. **[4]**

2 Find, to the nearest degree, the angle between the vectors $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix}$. **[3]**

3 Find real numbers a and b such that $(a - 3\mathrm{i})(5 - \mathrm{i}) = b - 17\mathrm{i}$. **[5]**

4 Find a cubic equation with real coefficients, two of whose roots are $2 - \mathrm{i}$ and 3. **[5]**

5 A transformation of the x-y plane is represented by the matrix $\begin{pmatrix} \cos\theta & 2\sin\theta \\ 2\sin\theta & -\cos\theta \end{pmatrix}$, where θ is a positive acute angle.

(i) Write down the image of the point (2, 3) under this transformation. **[2]**

(ii) You are given that this image is the point $(a, 0)$. Find the value of a. **[5]**

6 Find the invariant line of the transformation of the x-y plane represented by the matrix $\begin{pmatrix} 2 & 0 \\ 4 & -1 \end{pmatrix}$. **[4]**

7 **(i)** Express $\dfrac{1}{2r-1} - \dfrac{1}{2r+1}$ as a single fraction. **[2]**

(ii) Find how many terms of the series

$$\frac{2}{1\times 3} + \frac{2}{3\times 5} + \frac{2}{5\times 7} + \ldots + \frac{2}{(2r-1)(2r+1)} + \ldots$$

are needed for the sum to exceed 0.999 999. **[7]**

8 Prove by induction that $\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{pmatrix}$ for all positive integers n. **[6]**

9 Fig. 9 shows a sketch of the region OPQ of the Argand diagram defined by

$$\left\{z : |z| \leqslant 4\sqrt{2}\right\} \cap \left\{z : \tfrac{1}{4}\pi \leqslant \arg z \leqslant \tfrac{1}{3}\pi\right\}.$$

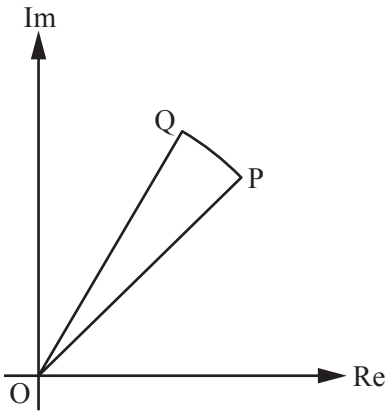

Fig. 9

(i) Find, in modulus-argument form, the complex number represented by the point P. **[2]**

(ii) Find, in the form $a + ib$, where a and b are exact real numbers, the complex number represented by the point Q. **[3]**

(iii) **In this question you must show detailed reasoning.**

Determine whether the points representing the complex numbers

- $3 + 5i$
- $5.5(\cos 0.8 + i \sin 0.8)$

lie within this region. **[4]**

10 Three planes have equations

$$\begin{aligned} -x + 2y + z &= 0 \\ 2x - y - z &= 0 \\ x + y &= a \end{aligned}$$

where a is a constant.

(i) Investigate the arrangement of the planes:

- when $a = 0$;
- when $a \neq 0$. **[6]**

(ii) Chris claims that the position vectors $-\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $2\mathbf{i} - \mathbf{j} - \mathbf{k}$ and $\mathbf{i} + \mathbf{j}$ lie in a plane. Determine whether or not Chris is correct. **[2]**

END OF QUESTION PAPER