OCR
Oxford Cambridge and RSA

# Wednesday 28 June 2017 – Morning

## A2 GCE MATHEMATICS

**4731/01** Mechanics 4

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4731/01
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

*6862814624*

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

**1** A uniform rod with centre $C$ has mass $2M$ and length $4a$. The rod is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through a point $A$ on the rod, where $AC = ka$ and $0 < k < 2$. The rod is making small oscillations about the equilibrium position with period $T$.

**(i)** Show that $T = 2\pi\sqrt{\frac{a}{3g}\left(\frac{4+3k^2}{k}\right)}$. (You may assume the standard formula $T = 2\pi\sqrt{\frac{I}{mgh}}$ for the period of small oscillations of a compound pendulum.) **[4]**

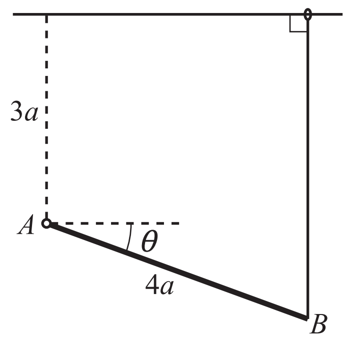
**(ii)** Hence find the value of $k^2$ for which the period of oscillations is least. **[3]**

**2** A ship $S$ is travelling with constant speed $5\,\mathrm{m\,s^{-1}}$ on a course with bearing 325°. A second ship $T$ observes $S$ when $S$ is 9500 m from $T$ on a bearing of 060° from $T$. Ship $T$ sets off in pursuit, travelling with constant speed $8.5\,\mathrm{m\,s^{-1}}$ in a straight line.

**(i)** Find the bearing of the course which $T$ should take in order to intercept $S$. **[4]**

**(ii)** Find the distance travelled by $S$ from the moment that $T$ sets off in pursuit until the point of interception. **[5]**

**3**




A uniform rod $AB$ has mass $m$ and length $4a$. The rod can rotate in a vertical plane about a smooth fixed horizontal axis passing through $A$. One end of a light elastic string of natural length $a$ and modulus of elasticity $\lambda mg$ is attached to $B$. The other end of the string is attached to a small light ring which slides on a fixed smooth horizontal rail which is in the same vertical plane as the rod. The rail is a vertical distance $3a$ above $A$. The string is always vertical and the rod makes an angle $\theta$ radians with the horizontal, where $0 \leqslant \theta \leqslant \frac{1}{2}\pi$ (see diagram).

**(i)** Taking $A$ as the reference level for gravitational potential energy, find an expression for the total potential energy $V$ of the system, and show that

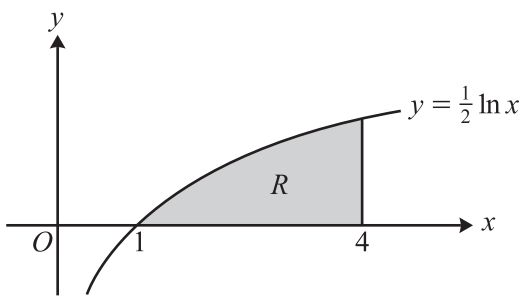
$$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 2mga\cos\theta\big(4\lambda(1+2\sin\theta)-1\big).$$ **[6]**

Determine the positions of equilibrium and the nature of their stability in the cases

**(ii)** $\lambda > \frac{1}{12}$, **[9]**

**(iii)** $\lambda < \frac{1}{12}$. **[2]**

**4**




The diagram shows the curve with equation $y = \frac{1}{2}\ln x$. The region $R$, shaded in the diagram, is bounded by the curve, the $x$-axis and the line $x = 4$. A uniform solid of revolution is formed by rotating $R$ completely about the $y$-axis to form a solid of volume $V$.

**(i)** Show that $V = \frac{1}{4}\pi(64\ln 2 - 15)$. **[4]**

**(ii)** Find the exact $y$-coordinate of the centre of mass of the solid. **[7]**

**5**

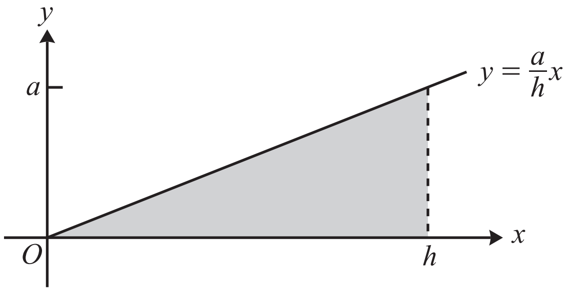



**Fig. 1**

Fig. 1 shows part of the line $y = \frac{a}{h}x$, where $a$ and $h$ are constants. The shaded region bounded by the line, the $x$-axis and the line $x = h$ is rotated about the $x$-axis to form a uniform solid cone of base radius $a$, height $h$ and volume $\frac{1}{3}\pi a^2 h$. The mass of the cone is $M$.

**(i)** Show by integration that the moment of inertia of the cone about the $y$-axis is $\frac{3}{20}M(a^2 + 4h^2)$. (You may assume the standard formula $\frac{1}{4}mr^2$ for the moment of inertia of a uniform disc about a diameter.) **[7]**

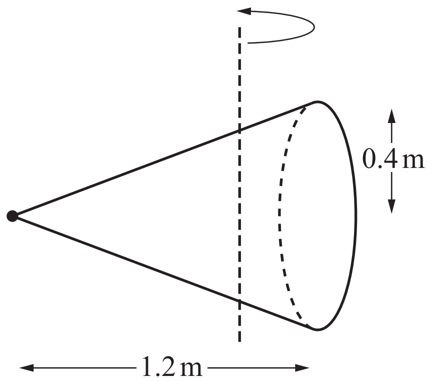
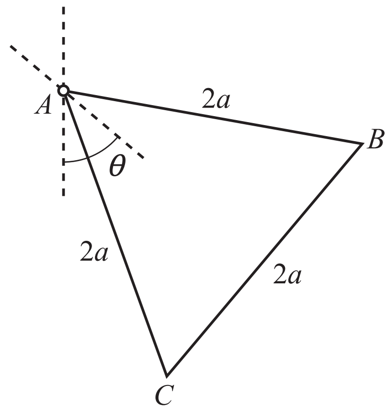



**Fig. 2**

A uniform solid cone has mass 3 kg, base radius 0.4 m and height 1.2 m. The cone can rotate about a fixed vertical axis passing through its centre of mass with the axis of the cone moving in a horizontal plane. The cone is rotating about this vertical axis at an angular speed of 9.6 rad s$^{-1}$. A stationary particle of mass $m$ kg becomes attached to the vertex of the cone (see Fig. 2). The particle being attached to the cone causes the angular speed to change instantaneously from 9.6 rad s$^{-1}$ to 7.8 rad s$^{-1}$.

**(ii)** Find the value of $m$. **[5]**

**6**




A triangular frame $ABC$ consists of three uniform rods $AB$, $BC$ and $CA$, rigidly joined at $A$, $B$ and $C$. Each rod has mass $m$ and length $2a$. The frame is free to rotate in a vertical plane about a fixed horizontal axis passing through $A$. The frame is initially held such that the axis of symmetry through $A$ is vertical and $BC$ is below the level of $A$. The frame starts to rotate with an initial angular speed of $\omega$ and at time $t$ the angle between the axis of symmetry through $A$ and the vertical is $\theta$ (see diagram).

**(i)** Show that the moment of inertia of the frame about the axis through $A$ is $6ma^2$. **[3]**

**(ii)** Show that the angular speed $\dot{\theta}$ of the frame when it has turned through an angle $\theta$ satisfies

$$a\dot{\theta}^2 = a\omega^2 - kg\sqrt{3}(1-\cos\theta),$$

stating the exact value of the constant $k$.

Hence find, in terms of $a$ and $g$, the set of values of $\omega^2$ for which the frame makes complete revolutions. **[5]**

At an instant when $\theta = \frac{1}{6}\pi$, the force acting on the frame at $A$ has magnitude $F$.

**(iii)** Given that $\omega^2 = \dfrac{2g}{a\sqrt{3}}$, find $F$ in terms of $m$ and $g$. **[8]**

**END OF QUESTION PAPER**

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